THE MECHANICS OF THE AEROPLANE

DUCHENE

TRANSLATED BY
JOHN H. LEDEBOER
AND
T. O'B. HUBBARD

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A STUDY OF THE PRINCIPLES OF FLIGHT

BY

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TRANSLATORS' PREFACE

Books on aeronautics may be roughly divided into two classes: the former written from an exclusively mathematical standpoint, and hence intended for but a small circle of readers; the latter, of a more elementary and popular character, do not, as a general rule, pretend to treat the problem of the aeroplane from its more serious technical and scientific aspects.

The present work belongs to neither of these categories. Its purpose is to explain in terms as simple as possible, and with a minimum of formulæ, the main principles of dynamic flight; to give the ordinary reader an insight into the various problems involved in the motion and equilibrium of the aeroplane; and to enable him to calculate in the simplest possible manner the various elements and conditions of flight.

At the outset of this work it may be well to provide against possible misconception by explaining that it in no way aspires to present in final and conclusive form the intricate problems which constitute the complete theory of the aeroplane—in view of the comparative youth of the science, such an attempt cannot be made for many years to come.

In consequence, the calculations it contains are approximate only; their numerical value, in fact, is founded on the basis of experiments so few in number that, even though their results be correct, they cannot well be accepted as final.

A few words of explanation in regard to the author's treatment of his subject may be required. In the first

place it is necessary to state—and the statement will be amply borne out by a perusal of the work—that throughout recourse has only been had to the simplest elements of mathematics and mechanics. All the mathematical knowledge required to follow the various arguments and calculations is, in fact, such as is possessed by almost every schoolboy.

The author, Captain Duchêne, is one of that brilliant band of French engineer officers whose contributions to the science of aeronautics have played a part of inestimable importance in the development of the aeroplane. Born in Paris on December 27th, 1869, he entered the Génie in 1890, after passing through the usual course at the École Polytechnique. He received his captaincy in November 1897, and was attached to the fortress of Toul, at that time one of the centres of military aerostation in France. Five years ago he was transferred to the staff at Paris.

The present work was awarded the Monthyon prize in 1911 by the Academy of Sciences. Although it may have lost in the process of translation some part of that lucidity and terseness of expression that form the most admirable and characteristic features of many French scientific works, we hope that the original value of Captain Duchêne's book remains unimpaired in its English form; that it may serve to correct much loose thinking and misapprehension at present prevailing, and that it may succeed in its endeavour to establish a firm connection between theory and practice.

J. H. L.
T. O'B. H.

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INTRODUCTORY

An aeroplane is an aerial machine sustained in the air by the support derived from its forward motion, which, therefore, is essential for it to remain aloft.

To produce this sustaining force advantage is taken of the established fact that the reaction of an air-current on a plane or any other surface is directed approximately at right angles to this surface; and this is true irrespective of the angle of incidence—that is, the angle at which the plane meets the air, which otherwise constitutes a factor of primary importance in the theory of the aeroplane.

It is evident that the property referred to holds good, whether an air-current strikes a fixed plane or, alternatively, whether the air-current be relative and produced—as in the case of the aeroplane—by the motion of the plane through still air.

From this consideration it follows that, by impelling horizontally a plane meeting the air at a very small angle,[1] it is possible to obtain from the relative air-current thus set up a reaction that is directed almost vertically upward; and if the velocity of motion is sufficiently high one may, by these means, succeed in raising and sustaining the weight of a complete aeroplane, with its planes, framework, motor, propeller, fuel, and passengers.

But the simple utilisation of the forward motion of an aeroplane to produce the requisite lifting-power only constitutes the first part of the problem. What is required for the solution of the whole problem is that the aeroplane should, in addition, be in equilibrium on its path of flight; and, further, that this equilibrium should be stable; that

[1] The angle of incidence of planes in practice never exceeds twelve degrees at the outside, and is, therefore, but little removed from the horizontal.

is, that the machine should not be liable to be upset or deflected from its course by the slightest disturbing influence.

These preliminary considerations indicate the lines along which the work has been subdivided.

The first, and longest, part deals with the support of the aeroplane in still air, apart altogether from any question of equilibrium or stability. In the first place, it treats of the principles of this sustaining-force, that is, with the action of a current of air on a plane. Hereupon follows an examination of the relations that connect, in horizontal flight, the speed, the thrust of the propelling mechanism, and the mechanical forces called into play with the important factor constituted by the angle of incidence and with the characteristics of the aeroplane, such as the weight, plane area, and "fineness." This first portion concludes with the consideration of the inclination of the flight-path and with the special case—undoubtedly the most interesting—of gliding flight [1]; some remarks will be made regarding starting and alighting.

The second portion is devoted to the consideration of the equilibrium and stability of the aeroplane in still air, in the threefold aspect of longitudinal, lateral, and directional stability. Turning is another subject for consideration.

The third portion of this work treats of the effect of wind on the aeroplane. The influence of a regular wind on the flight-path forms the first heading, and is followed by the effect on the equilibrium and stability of the aeroplane of irregular wind-currents, of atmospheric pulsations, and of gusts.

A final appendix treats of the design of screw-propellers.

[1] The terms "gliding flight," "glide," are used throughout to denote flight with the motor stopped, designated in French by the words "vol plané."—*Translators.*

ERRATA

P. 53, l. 18, *for* 'increasing' *read* 'testing.'
P. 75, l. 12, *for* 'one-fifth' *read* 'one-fiftieth.'
P. 202, l. 6, *for* 'rational' *read* 'rotational.'
P. 231, *for* '$s \ldots = \frac{P}{f^3 i}$,' *read* '$s \ldots = \frac{P}{f^2 i}$.'

THE MECHANICS OF THE AEROPLANE

PART I

FLIGHT IN STILL AIR

CHAPTER I

SUPPORT IN THE AIR—ACTION OF A WIND-CURRENT ON A PLANE

1. Action of a wind-current striking a plane surface at right angles.

A wind-current of velocity V (in metres per second), striking at right angles a plane surface AB of area S (in

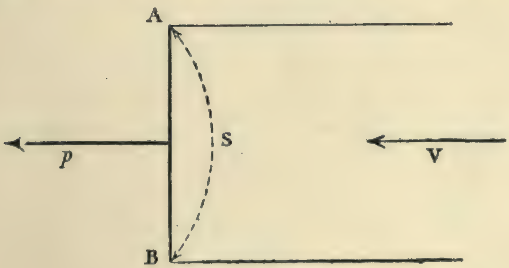

FIG. 1.—Profile.

square metres)—see Fig. 1—exerts on this plane an action equivalent to that of a force or pressure p similar in its

direction and effect to the wind-current and of the magnitude (in kilogrammes)[1]

(1) $$p = 0{\cdot}08\ SV^2.$$

Thus, it will be seen that *the magnitude of the pressure is proportional to the area of the plane and to the square of the velocity of the wind.* Consequently, the pressure grows very rapidly with an increase in speed.

It should be noted, in addition, that the value of this pressure is scarcely at all affected by the shape, in plan-form, of the plane, which may be square, rectangular, circular, &c.

And finally, it should be remembered that, as has already been stated, this value remains the same whether the wind be real—whether, that is, it strikes a plane fixed in position—or whether it be merely relative, as in the case of a plane moving through still air.

2. Action of a wind-current striking a plane surface at an angle.

A wind-current striking, at velocity V, at a small angle

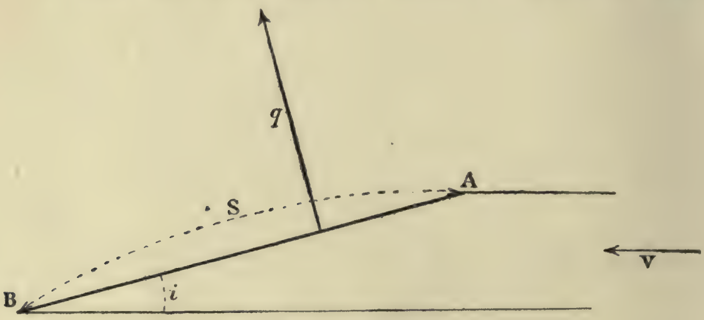

FIG. 2.—Profile.

i, a plane surface of area S exerts on this plane an action

[1] This value of the coefficient is approximately the mean value found by various experimenters, but it is not impossible that further experiments may, in the future, lead to its modification.

equivalent to that of a force or pressure q directed almost at right angles to the plane, and of the magnitude (in kilogrammes)

$$(2) \qquad q = kSV^2 i.$$

The symbol k represents a quantity that may be considered as a constant coefficient, provided that the planes acted upon are of similar shape and that the angle i remains small.[1]

The angle of incidence i is expressed as a decimal fraction, giving the slope of one of its sides relatively to the other: thus, an angle of 0·07 has one of its sides at a slope of 7 centimetres per metre relatively to the other.[2]

To sum up, just as in the case of the plane struck at right angles:

The pressure is proportional to the area of the plane and to the speed of the wind. In addition, it is proportional to the angle of incidence for small angles.

On the other hand, it differs from the plane struck by the wind at right angles, in that both the shape of the plane and its position relatively to the wind-current greatly affect the quantity of the pressure.

The pressure, in fact, increases as the span of the plane —that is, its dimension transverse to the direction of flight —is increased relatively to its fore-and-aft dimension. The relation between span and the fore-and-aft dimension is known as the "aspect ratio."

This phenomenon, as a matter of fact, admits of a simple explanation. It is, in fact, evident that the tendency of the air to leak over the sides of the plane remains the same in the case of the square plane ABCD as in the case of the wide-span plane A′B′C′D′, and this "leakage" naturally affects the total pressure to a lesser extent, as this pressure increases, than in the case of a wide-span plane.

[1] See footnote to p. 4.

[2] For small angles this is equivalent to the sine of the angle.—*Translators.*

And, if we pursue the same line of argument, it is also clear that the advantage of the increase in span must gradually grow less as the span is increased. In practice nothing is gained by a greater aspect ratio than 5 or 6

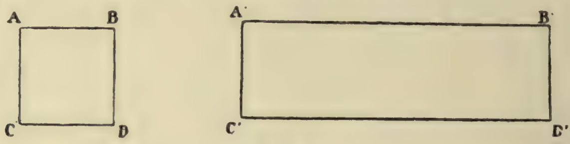

FIG. 3.

to 1. As a matter of fact certain devices—such as the cellular partitioning of the early Voisin biplanes—even render it unnecessary to reach this limit.

It is safe to accept the increased pressure due to a good aspect ratio as being about 1·6 times that of the pressure on a square plane of equal area.

Accordingly, in formula (2) the value of the coefficient k may be modified so as to allow for the effect of aspect ratio.

For a square plane, for instance, the old formula of Duchemin gives this coefficient a value double that of 0·08, its value when struck at right angles—that is, 0·16. In the case of a good aspect ratio, this value is increased 1·6 times, and is thus raised to 0·26. *These values, however, are only given by way of example, and should not be accepted literally.*[1]

This influence of the aspect ratio deserves the closest attention: the wide span of a bird's wings forms the best natural illustration of the principle.

[1] As a matter of fact, M. Eiffel's recent researches give different values to this coefficient. For a flat plane of 5 or 6 aspect ratio, its value would seem to be about 0·34 when the angle of incidence is less than 0·13 7½ degrees). As the angle of incidence increases beyond this figure, the value of k rapidly falls. The variation in the value of the pressure relatively to the angle of incidence, as established by M. Eiffel's experiments, may be represented in the following two formulæ:

If i is less than 0·13 . . . . . . $q = 0{\cdot}34\ SV^2 i$.
If i is between 0·13 and 0·25 . . $q = (0{\cdot}04 + 0{\cdot}02i)\ SV^2$.

The same principle leads to the rejection *a priori* of such aeroplanes as are sometimes designed to fly with their smaller dimension transverse to the line of flight, whereof the schoolboy's paper dart forms a good example. Such planes must, in fact, be most inefficient owing to the great leakage of the air over the sides.

3. Action of a wind-current striking at an angle a curved plane.

Modern theory and practice have shown that the lifting efficiency of a plane is greatly increased by curving it longitudinally, the concave surface being placed so as to meet the air. Such curved planes are seen in a bird's wings.

The advantageous effect of thus curving the plane (a second advantage will be explained in § 11) resides in the fact that the pressure on a curved plane is considerably greater than that on a flat plane of equal area and struck by the wind at the same angle.

But in order to justify any comparison, it here becomes necessary to define precisely what constitutes the angle of incidence of a curved plane.

At first one is naturally inclined to consider the chord of the curve as the angle of incidence of the plane. Nevertheless, as will be seen, the line of the chord would not furnish a correct basis for comparison of the pressures on flat and curved planes respectively. It is, in fact, clear that, when a flat plane meets the air with its leading edge edge-on, no lift of any kind is produced; in other words, it sets up no reaction directed perpendicularly to the air-current. When, on the other hand, a curved plane meets the air edge-on, there arises a distinct lift.

So that there may be no lift, it is necessary that the curved plane should be struck by the air-current slightly on its upper surface, at an angle shown in Fig. 4 by the line A′C. And the angle of incidence, according to M. Soreau, should be reckoned from this line.

Adopting this angle as the angle of incidence, we may now establish the following important formula on analogous lines to formula (2):

$$(3) \qquad Q = KSV^2 i,$$

wherein Q represents the pressure (in kilogrammes) exerted by the air on the plane, S represents its area in square

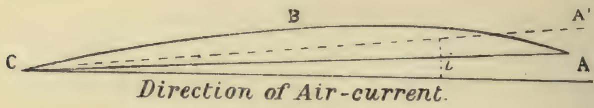

FIG. 4.—Direction of Air-current.

metres (no regard being paid to the curvature), V the velocity in metres per second, and i the angle of incidence expressed as a decimal fraction and calculated from the base-line as shown. Finally, the symbol K is a coefficient that remains constant so long as the angle of incidence remains small.

If, on the other hand, the angle of incidence were calculated from the chord of the curve, formula (3) could only be made to apply if the coefficient K varied with the angle. As a special case: If the angle of the chord were zero—that is, if the direction of the air-current were along the chord—the coefficient K would become infinitely great in value (for, although the angle i would be zero, the pressure Q would nevertheless be appreciable). If, however, the chord is calculated to form the angle of incidence —as is sometimes the case—the value of K can only be regarded as constant for very small variations in this angle.

Clearly, the values of K and of the angle formed by the line that constitutes the base-line of the angle of incidence and the chord will depend on the shape and depth of the curve.

Captain Ferber laid down as the most efficient shape a curve with a greatest depth (known as the "camber") of $\frac{1}{15}$th

of the fore-and-aft dimension of the plane and situated at ⅓rd of this dimension from the forward edge (Fig. 5).

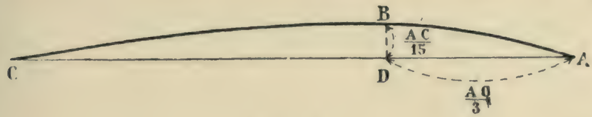

FIG. 5.

The distinctly flat curve adopted by the brothers Wright as the result of long experiment, and approximately shown in Fig. 6, would appear to have given the best practical results hitherto.

FIG. 6.

The foregoing considerations render it evident that a certain amount of indecision still prevails regarding the value to be given to the coefficient K, and that different authors, in fact, have reached widely varying results. Captain Ferber, for instance, gives the value as 0·7, whereas M. Soreau fixes it at 0·4.[1]

But in any case it is clear that the coefficient K, provided it is defined with sufficient accuracy, will be the

[1] According to M. Eiffel's more recent experiments with a curve forming the segment of a circle and with a camber of $\frac{1}{13\cdot5}$ of the chord situated at the centre of the curve, for angles below 0·35 (the base-line being reckoned as forming an angle of 0·15 with the chord) the coefficient K has a value of 0·225. In other words, if i' represents the angle formed by the direction of the air-current with the chord, the pressure on the plane, for angles between 0 and 0·20 could be calculated from the formula

$$Q = 0{\cdot}225\ SV^2\ (i' + 0{\cdot}15). \tag{3A}$$

But it should also be noted that the curve with which the above experiments were carried out was exceptionally deep. Probably with flatter curves, such as that used by the brothers Wright, the base-line of the angle of incidence would form a smaller angle with the chord than 0·15, so that the coefficient K would approximate to the value 0·4 given it by M. Soreau.

higher, the better the depth of the camber and the aspect ratio of the plane selected. Consequently, the coefficient K characterises the *lifting efficiency* of a plane, and will therefore be designated as such hereafter.

Example of application of formula (3).

What is the amount of the pressure exerted on a plane, of efficient camber and aspect ratio, which has a lifting efficiency of 0·4 and an area of 25 sq. m., by an air-current of 20 m. p. sec. striking it at an angle of incidence of 0·12 ?

The value of Q in kilogrammes will be:

$$Q = \overset{K}{0{\cdot}4} \times \overset{S}{25} \times \overset{V^2}{400} \times \overset{i}{0{\cdot}12} = 480 \text{ kg.}$$

The table of squares on page 222 will facilitate the rapid calculation of such problems.

4. Equivalent flat plane—Composition of the planes.

From the foregoing it will be understood that it is possible, for purposes of calculation, to substitute for any curved plane an imaginary flat plane of the same area but having a lifting efficiency K equal to that of the curved plane, and being inclined at an angle equal to that of the base-line of the angle of incidence of the curved plane.

This method of calculation may be extended still further by substituting for all the planes of a whole aeroplane a single imaginary flat plane of equal area to the total area of all the real planes and having a mean lifting efficiency K.[1] M. Soreau has given the title of *equivalent flat plane* of an aeroplane to the imaginary plane calculated according to this method of *composition of the planes*.

[1] In the case of planes arranged in parallel, if $K_1, K_2 \ldots K_n$ represent the lifting efficiencies of the planes of area $S_1, S_2 \ldots S_n$, the value of K is derived from the expression

$$\frac{K_1 S_1 + K_2 S_2 + \ldots + K_n S_n}{S_1 + S_2 + \ldots + S_n}$$

CHAPTER II

HORIZONTAL FLIGHT OF AN AEROPLANE IN STILL AIR

I.—The Basic Formula

5. Condition necessary for flight.

In the first place, it will be remembered (see Introductory Chapter), that in flight an aeroplane assumes a position of stable equilibrium on its path of flight, which it follows at a constant angle of incidence to the relative wind-current set up by its forward motion.

The pilot is able to vary this angle of incidence by manipulating the *elevator*, whose action will be described in detail at a further opportunity (§ 52, p. 117).

In order that an aeroplane may be sustained in the air it is necessary that the pressure exerted on its equivalent flat plane by the wind-current that meets it should have a vertical component equal to the weight of the aeroplane.

In Fig. 7, let AB be the equivalent flat plane of the aeroplane, and Q the pressure exerted on it by the air. So long as the angle of incidence i remains small, the direction of this pressure is but little removed from the vertical. The vertical pressure component F may therefore, without any appreciable error, be supposed equal in magnitude to the total pressure Q. The vertical component F is termed the "lift."

If P represents the weight of the aeroplane in kilogrammes, the necessary condition for flight is represented by the equation $P = Q$.

But the value of Q may be found from formula (3)—

$$Q = KSV^2 i,$$

where the symbols i, V, S, K represent respectively the angle of incidence, the speed, the area of the equivalent flat plane, and its lifting efficiency.

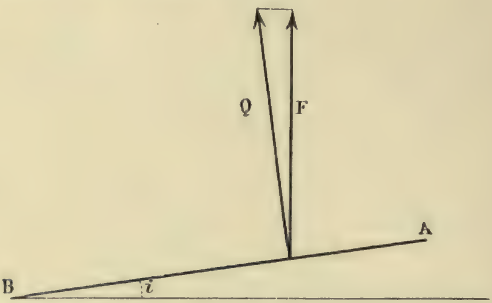

FIG. 7.

Hence we may evolve the basic formula which, though simple in form, represents algebraically the important condition for flight of an aeroplane:

$$P = KSV^2 i. \tag{4}$$

II.—THE SPEED

6. The speed at which an aeroplane travels in horizontal flight.

From formula (4) may be deduced:

$$V = \sqrt{\frac{P}{KSi}} \tag{5}$$

which expresses the relation connecting the speed of the aeroplane in horizontal flight with the following factors:

The angle of incidence i of the equivalent flat plane,
The weight P of the aeroplane,
The area S,
The lifting efficiency K of its planes.

Of these four quantities only the first, the angle of incidence i, is at the present time capable of being varied during the course of flight.[1] The last three quantities may henceforward be termed the *characteristics* of the aeroplane.

To begin with, it is therefore necessary to examine the influence exerted by a variation of the angle of incidence on the normal speed of an aeroplane in horizontal flight. The three characteristics—weight, area, and lifting efficiency—are fixed.

The next step is to examine the effect on the horizontal speed of an alteration in any one of the three fixed characteristics—the third characteristic and the angle of incidence remaining constant.

It should be distinctly understood that this latter case no longer deals with an alteration in the speed of a given aeroplane by the manipulation during flight of such an organ as the elevator, but is confined to the comparison of the speeds of two aeroplanes that only differ in a single characteristic (either in weight, area, or lifting efficiency), other things remaining equal and the angle of incidence being identical in each case.

A third and final aspect of the problem will be examined in §§ 9 and 31.

7. Variation in the speed of a given aeroplane caused by the alteration of the angle of incidence—Influence of motive power.

On examination of formula (5):

$$V = \sqrt{\frac{P}{KSi}}$$

it is clear that in a given aeroplane where P, S, and K are constant in value:

The speed in horizontal flight depends only on the angle of incidence.

[1] It is possible that, in course of time, new inventions will give the pilot the ability to vary the area or the lifting efficiency of his planes during the course of flight.

At first sight this extremely important fact is likely to cause surprise. For it would appear that the speed of a motor-propelled vehicle depends entirely on the mechanical power developed by the motor; wherefore, by increasing the power, the speed would also increase.

The fact is certainly true in so far as any land vehicle is concerned, for it is forced to remain on the earth's surface, and its movements are consequently confined to two dimensions. The aeroplane, on the other hand, is capable of free motion through the three-dimensional space of the atmosphere. Its motive power may affect the speed to some extent, but only indirectly and through the intermediary of an element peculiar only to the aeroplane, and without an equivalent in any other vehicle of locomotion. That element is the angle of incidence.

Now, it is evident in the first place that, in order to produce, on the planes of an aeroplane meeting the air at a certain angle, a pressure whose vertical component or lift shall balance the weight of the machine, the aeroplane must move at a certain given speed, and at no other. If the speed is either greater or less than the given speed, the aeroplane must either rise or fall.

Further, it is clear that if the angle of incidence is altered, then the one speed necessary for horizontal flight will no longer be the same as in the first case.

Every value of the angle of incidence therefore requires one definite rate of speed, as has been deduced from formula (5).

What, then, is the part played in horizontal flight by the power developed by the motor? On a later page (§ 43, p. 92) will be shown that the sole part played by the motor is to maintain flight horizontal, to prevent the machine from falling under the action of gravity.

If, during the course of horizontal flight, the pilot stops his engine *without interfering with the elevator*, the aeroplane starts to glide; that is, it follows a slowly

descending flight-path, but—and this is the important point—**its velocity of flight remains practically the same as before.**

It was not, therefore, the motive power that created and fixed the speed, for this remained unaltered when the motor was entirely stopped.

The factor that governed the speed was the angle of incidence, a factor depending wholly on the relative disposition of the planes and other parts of the machine, and particularly on the position of the elevator.

If, as was supposed, the pilot did not manipulate his elevator, the aeroplane assumed on its inclined gliding path precisely the same position of equilibrium that it possessed while in horizontal flight. Both in its former and latter position, therefore, the angle of incidence of the planes to the air-current created by its own speed remained the same. And this clearly shows why the speed also remained the same.

FIG. 8.

If, while gliding, the pilot re-started his motor, gradually increasing the power, the flight-path would gradually rise to and even beyond the horizontal. **But if the elevator remained untouched, that is, if the angle of incidence were not varied, the speed of the aeroplane would always remain the same**. From this it will be seen that, as already stated, the one function of the motive power is to overcome or moderate the action of gravity, that is, to regulate the flight-path vertically, without interfering directly with the speed of the aeroplane.

At the same time, it would be a mistake to conclude

that it is impossible to increase the speed of an aeroplane by increasing the motive power. For, if the only effect would be to impart an upward tendency to the flight-path so long as the elevator remains untouched, this no longer holds true when the elevator is manipulated and the angle of incidence thereby modified.

In the latter case the speed of horizontal flight assumes, for each different value of the angle of incidence, the one corresponding value which may be deduced from formula (5).

It then becomes possible to find the correct position of the elevator, that is, the correct angle of incidence, that will cause the increase in the motive power to be transformed, not in raising the flight-path, but in increasing the speed of the aeroplane which then remains in horizontal flight.

A variation in the power can, therefore, affect the speed, but only in the case where the angle of incidence is modified.

In other words, as already stated:

The motive power can only affect the speed of flight through the intermediary of the angle of incidence.

Although rather anticipating questions that will be dealt with later on, this somewhat lengthy explanation was deemed advisable, in view of the necessity for clearly grasping the main feature of the horizontal flight of an aeroplane, which is totally different from all other vehicles in that it alone possesses the power of vertical motion.

In practice, as a matter of fact, the motor usually runs at a certain constant speed, so that the pilot has to find by experiment the single position for the elevator that will give the angle of incidence the correct value where the total power developed is absorbed usefully.

The speed of the aeroplane then becomes that which corresponds to that particular value of the angle of incidence. This rate of speed is termed the *normal speed*

of the aeroplane, and the corresponding angle of incidence is termed the *normal angle of incidence.*

8. Speed curves—Attainable speeds.

If in formula (5),

$$V = \sqrt{\frac{P}{KSi}},$$

where the characteristics P, K, and S are supposed constant, the angle of incidence i is given various values; one

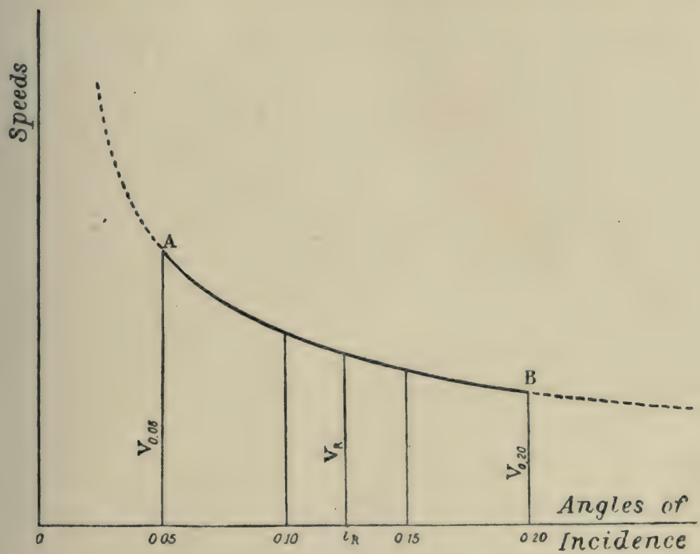

FIG. 9.

obtains the corresponding values of the speed V, which can be plotted out in a curve somewhat of the shape shown in Fig. 9.

The first result is the proof that:

When the angle of incidence increases, the speed diminishes, and vice versa. Secondly, the fact that the curve is concave in its upper portion shows that the variation in the speed corresponding to an equivalent variation

of the angle of incidence is greater in proportion as the latter is smaller. As an instance, the increase in speed is greater, if the value of the angle is diminished by 0·01, in the case where the original value of the angle was 0·08, than if it had originally been 0·15.

Accordingly, the use of a very small angle of incidence would seem to render possible the attainment of high speeds. But such speeds would only be possible for an enormous expenditure of power;[1] and further, the diminution of the angle of incidence is limited by the danger that the planes of such an aeroplane might be struck by the wind on their upper surface at the slightest longitudinal oscillation.

Practically, therefore, the angle of incidence cannot be reduced beyond a strict limit which varies according to the type of aeroplane. Speaking in general terms, this extreme limit may be fixed at 0·05; that is about 3°.[2]

But, on the other hand, it has been stated (§§ 3 and 5) that the fundamental formula (4) was only applicable for small angles of incidence. According to M. Soreau, this angle in practice is always less than 0·20 (about 12°). This shows that the curve derived from formula (5), which in its turn is derived from formula (4), need only be considered in the portion between points A and B, which indicate the angles 0·05 and 0·20 respectively.

By *attainable speeds* is meant the speeds comprised by this portion of the curve. Not that an aeroplane can necessarily attain all such speeds, but only that the speed it can attain must perforce be included in this relatively small portion of the curve between the limits $V_{0·05}$ and $V_{0·20}$ of which the first is just about double the second.

The normal speed V_R, as stated previously (§ 7, p. 14), is

[1] See also § 21, p. 42.

[2] In all probability such a low angle has never yet been obtained in practice.

in practice determined by the motive power, which can only be wholly absorbed in horizontal flight at a certain definite angle of incidence. This normal speed V_R corresponds to the normal angle i_R which must necessarily be greater than the lower critical angle. From this it follows that the normal speed is bound to be included not only among the attainable range of speeds, but that it must be lower than the speed corresponding to the lower critical angle (that is, to $V_{0·05}$).[1]

From this it is evident that it would be dangerous to apply to a given aeroplane motive power which would necessitate the employment of too small an angle of incidence. Wherefore, the motor must necessarily be proportioned to the aeroplane, because danger would arise from the employment of too powerful a motor in an aeroplane designed for less power.

Nevertheless, it may be pointed out that under certain circumstances this procedure may be adopted (see § 25), but in this case the motor only gives a portion of its power in normal flight.

In practice the aeroplane always flies at its normal speed, and its various parts must be designed and disposed with a view to this speed.

If the pilot wishes to fly horizontally at a different speed from the normal speed, he is obliged to vary, not only the angle of incidence by operating the elevator, but the power developed by the motor.

9. Effect of the value of the aeroplane's characteristics on its speed.

The next step in considering formula (5)

$$V = \sqrt{\frac{P}{KSi}}$$

is to deduce therefrom the variations to which is subject

[1] This argument is considered in greater detail in §§ 25, 27, and 30.

B

the speed when any single one of the aeroplane's characteristics P, S, or K is modified, while the other two, as well as the angle of incidence i, remain constant.

In the first place, it is to be noted that the value of the speed is proportional to

$$\sqrt{\frac{P}{S}}.$$

If the term *loading* is applied to the quantity $\frac{P}{S}$ (in kilogrammes per sq. metre), one can write:

Other things being equal, the speed is proportional to the square root of the loading of the planes.

As the loading remains an invariable quantity of the weight of the aeroplane, and its plane area either increases or diminishes in the same proportion, it follows that the speed also remains invariable. But if only one of these two characteristics varies—the other remaining constant—the speed must also vary.

An increase in the weight of the aeroplane brings about an increase in speed proportional to the square root of the ratio of increase, and vice versa.

Any reduction of the plane area also causes an increase in the speed proportional to the square root of the ratio of reduction, and vice versa.

Should, for instance, the weight of the aeroplane be quadrupled, while the other characteristics (area and efficiency) and the angle remain constant, the speed will be doubled. The same result would follow if the plane area were reduced to one quarter.

Thus one is justified in assuming that, in the event of high-speed machines being built in the future, their weight will be considerable, or, more probably, their area will be small.

It will be shown hereafter that such high-speed machines will require very high-powered engines (see § 22); their second disadvantage is the enormous diffi-

culty in starting and alighting. But it is not impossible that these difficulties may be eventually overcome by the invention of the variable-surface machine, which would permit a high speed to be maintained in normal flight, while starting and landing could be accomplished at slow speed.

In concluding the discussion of formula (5) a few remarks may be made on the effect on the speed of the lifting efficiency K of the planes.

Examination shows that if the other characteristics (weight, speed, area) and the angle remain constant, the speed is inversely proportional to the square root of K; which produces the—at first sight—astonishing result that, other things being equal:

The aeroplane possessing the most inefficient planes will fly fastest.

A child's paper dart forms a good illustration of this principle. But it will be shown at a later stage (§ 22) that this advantage is only apparent, and is only obtained at the cost of excessive motive power. Here again, the future may solve the problem of the variable-speed machine by enabling the plane efficiency to be reduced in flight so as to increase the speed.[1]

Throughout the preceding discussion the angle of incidence has been supposed to remain constant. But at this point it is necessary to explain more precisely the exact meaning of this assumption. It must, in fact, be understood that, whether we compare the respective speeds in flight, at a constant angle, of two separate aeroplanes exactly similar except for a single one of their characteristics, or whether we compare the speed of a single aeroplane before and after obtaining one of its characteristics, *the motive-power must in each case be precisely sufficient for the conditions of flight.*

[1] The use of planes with a variable camber, for instance, would, if satisfactory in practice, allow the plane efficiency to be varied in flight.

In other words, we have had to assume that, in comparing two aeroplanes, the power in either case is sufficient for the purpose of its flight, and that, in the case of a single aeroplane, the pilot is able—by some means or other—to make his engine develop the different powers required for horizontal flight at a fixed angle of incidence in accordance with the variation in the value of the characteristic modified.

But the whole problem is changed if the value of any one characteristic is modified on an aeroplane driven by an engine that normally develops its full power (always running "all out"), for this power remains practically constant at every speed, as will be shown below (see § 29).

Now, although this problem is interesting enough in itself, it can obviously find no place in the preceding discussion; for it necessarily entails—owing to the variation of one characteristic—a consequent variation in the angle of incidence. The latter, therefore, no longer remains constant.

This important distinction is often ignored, and the result is confusion. Take the case of the effect produced on the horizontal speed of a machine by the carrying of an extra passenger. *If the angle remained constant* the increase in the weight would entail an increase of speed. But this would further require an increase in power. On the other hand, if the motor were already producing its full power, *the angle of incidence would have to be increased*, which would result in a reduction of the speed.[1]

These two results are obviously contradictory, which is due to the fact that the two problems set were radically different.

Reference was made to the latter of the two at the end of § 6; it will be examined at greater length in § 31.

[1] This explanation is only approximately correct; the real reason for the reduction of the speed will be given in § 31.

10. Tables for the rapid calculation of speeds in horizontal flight.

The two following tables, based on formula (5), enable an immediate calculation to be made of the speed at which flies in horizontal flight an aeroplane of a given weight P, of area S, and flying at a small angle of incidence i.

Table I. gives the speeds corresponding to various angles of incidence, the loading being taken at 10 kg. per sq. metre.

Table II. gives the numbers by which it is necessary to multiply the speeds given in Table I., according as the loading varies from 10 kg. per sq. metre.

These Tables are based on the value 0·4 obtained by M. Soreau[1] for the coefficient K or lifting efficiency of a good plane. Their accuracy consequently depends on that of this value of the coefficient. It must here be once more repeated; that the quantities given in these and other Tables must not be taken to be accurate. They are merely approximations.

TABLE I.

Speeds in horizontal flight at varying angles of incidence, with a loading of 10 kg. per sq. metre.

Angle i. . . .	0·05	0·06	0·07	0·08	0·09	0·10	0·11	0·12
Speed in m/s. .	22·36	20·41	18·90	17·68	16·67	15·81	15·08	14·45
Angle i. . . .	0·13	0·14	0·15	0·16	0·17	0·18	0·19	0·20
Speed in m/s. .	13·87	13·36	12·91	12·50	12·13	11·79	11·47	11·18

[1] If the coefficient K be given a different value K′, the figures giving the speeds in Table I. only require dividing by $\sqrt{\frac{K'}{0{\cdot}4}}$. Table II. would not require modification.

TABLE II.

Numbers by which must be multiplied the speeds given in Table I. according to different loadings.

Loading in kg. per sq. m. . .	4	5	6	7	8	9	10	11	12
Multipliers. . .	0·632	0·707	0·775	0·837	0·894	0·949	1·000	1·049	1·095
Loading in kg. per sq. m. . .	13	14	15	16	17	18	19	20	21
Multipliers. . .	1·140	1·183	1·225	1·265	1·304	1·342	1·378	1·414	1·449
Loading in kg. per sq. m. . .	22	23	24	25	26	27	28	29	30
Multipliers. . .	1·483	1·517	1·549	1·581	1·612	1·643	1·673	1·703	1·732

An example of how to use these Tables.

Calculate the speed in horizontal flight at an angle of incidence 0·13, of an aeroplane weighing 480 kg. with a plane area of 40 sq. m.

The loading will be $\frac{480}{40}=12$ kg. per sq. metre. The figure corresponding to this loading in Table II. is 1·095. The speed is then found by multiplying by 1·095 the speed (13·87 m/s) corresponding in Table I. to the angle 0·13.

The exact speed therefore will be $13{\cdot}87 \times 1{\cdot}095 = 15{\cdot}19$ metres per second.

III.—DRIFT

11. Resistance to forward motion—Drift.

In Fig. 10, let $ON = R$ be the reaction of the air on an aeroplane travelling horizontally at an angle of incidence i and at its proper speed of travel.

This reaction R is made up not only of the pressure Q exerted on the planes; it must be reckoned to be the resultant of the action of the air in flight on every part: planes, frame, body, landing-gear, &c.

The distinction is important, for whereas the vertical component or lift P = OM, which is equal to the weight of the aeroplane, is practically the same as the total pressure Q, the parts of the aeroplane other than the planes not exerting any lift, the case is different with the hori-

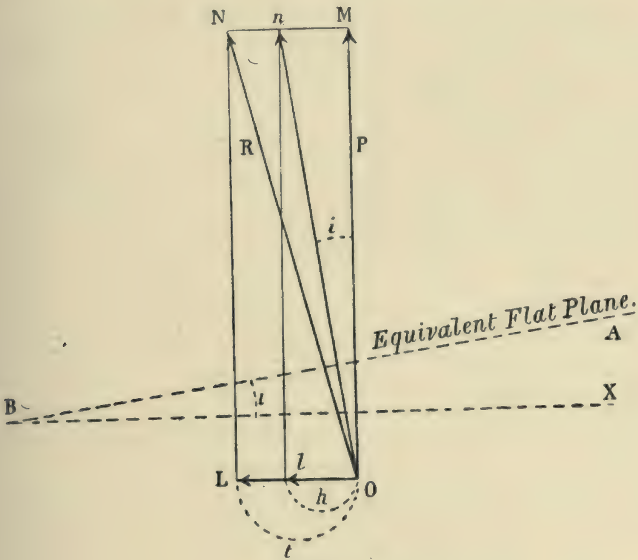

FIG. 10.

zontal component t = OL, which is much greater than the horizontal resistance of the planes alone.

This component represents the reaction horizontally opposed by the air to the forward motion of the aeroplane, and is known as the resistance to forward motion or *drift* of the aeroplane.

The function of the motor and propeller[1] is to create

[1] Motor and propeller together are termed the *propelling system*, but, as a matter of fact, the term *lifting system* would be more appropriate; for, as has been shown in § 7, the forward motion of an aeroplane is not the result of the action of the motor, since it persists when the latter is stopped. The propelling system, therefore, rather lifts than propels the aeroplane.

horizontally in the direction of flight a force, termed the *thrust*, which must, in accordance with the principle of the equality of action and reaction, be equal in magnitude to the drift. Consequently: **the thrust is equal and directly opposed to the drift.**

If the total reaction R on the aeroplane were accurately perpendicular to the equivalent flat plane and directed along On, its horizontal component h would be equal to Ol or to Mn. Now, since the angle MOn is equal to the angle of incidence ABX or i (for their respective sides are at right angles to one another), and since the angle i may be expressed as a decimal fraction, Mn may be taken as equal to OM $\times i$, that is, to Pi.

We may therefore write:

$$h = \mathrm{P}i. \tag{6}$$

The first portion h of the drift is due to the inclination of the plane, which, of course, is essential to produce the necessary lift. Thus, this portion h of the resistance to forward motion may be described not altogether inaptly as the price paid for the lift, since the latter can only be obtained by overcoming the former. For this reason it may also be termed the *active resistance.*

It diminishes as the angle of incidence grows smaller, as may be seen from formula (6). If, therefore, it were possible to fly at a very small angle indeed, that is at a very high speed, this part of the drift would become negligible.

This is clearly shown by writing formula (6) in the form [1]

$$h = \frac{\mathrm{P}^2}{\mathrm{KSV}^2}, \tag{6a}$$

so that, in a given aeroplane, the active resistance is smaller according as the speed is higher.

[1] Since $\mathrm{P} = \mathrm{KSV}^2 i$, or $i = \frac{\mathrm{P}}{\mathrm{KSV}^2}$.

Hence it follows that if active resistance made up the whole of the drift or resistance to forward motion, the thrust required would be the smaller the higher the speed. But unfortunately such is not the case, for to the active resistance must be added the *passive resistance* or *head resistance*,[1] caused by the thickness of the planes, the friction of the air on their surfaces, the presence of the framework with its uprights, the stay wires, the landing chassis, the motor, and the pilot with his passenger.

The head resistance is considerably diminished by the use of curved instead of flat planes. For, in addition to increasing the lift, a curved plane has the advantage of causing the pressure exerted on a plane by a horizontal air-current to assume a more nearly vertical direction. In fact, for certain angles of incidence, the direction of the pressure may actually pass the perpendicular to the equivalent plane. According to M. Eiffel's latest experiments, it may even, for certain angles of incidence, pass further beyond the vertical than the perpendicular to the chord. (See Fig. 11.)

The use of curved planes therefore brings about, to adopt M. Soreau's expression, a counter-resistance to forward motion, or "negative" resistance, which diminishes (without, however, entirely nullifying it) the head resistance set up by the parts of the aeroplane other than the planes.

The total result of these various sources of resistance, diminished as it is by the negative resistance due to the curving of the planes, may be considered as if it were caused by the effect of a single surface such, for instance, as a disc placed at right angles to the direction of flight. This imaginary surface is known as the *detrimental surface* of the aeroplane.

Its value is one of the characteristics of the machine, together with the weight, plane area, and lifting efficiency.

It is usually represented by the symbol s, and the

[1] The latter term is usually employed in English.—*Translators.*

value of p may be expressed, according to formula (1)—see § 1—as follows:

$$(7) \qquad p = 0{\cdot}08 s V^2.$$

Therefore, whereas the active resistance diminishes proportionately to the speed, the head resistance grows proportionately to the speed.

This explains the universal aim in aeroplane design to reduce to the lowest possible degree every cause of head

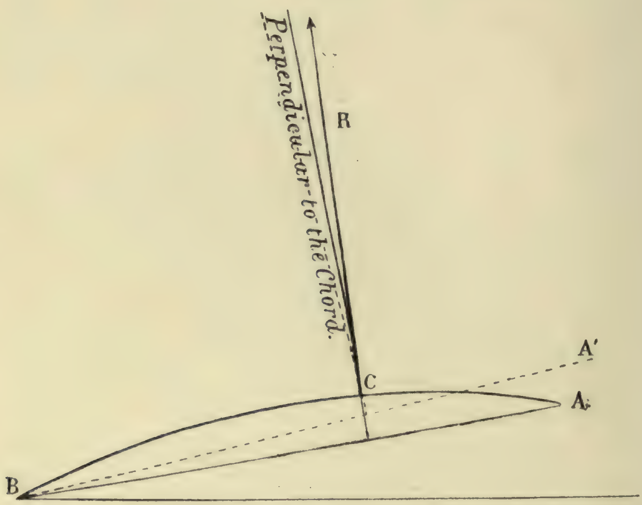

FIG. 11.

resistance, by curving the planes, "shaping" every exposed part, by the occasional use of wind-shields, by eliminating wires, &c.

To sum up, the total resistance to forward motion, or drift, of an aeroplane, or the thrust—which is its equivalent—may be written:

$$(8) \qquad t = h + p = Pi + 0{\cdot}08 s V^2,$$

or, replacing V^2 by its value $\frac{P}{KSi}$, according to the basic formula (4):

$$t = Pi + \frac{P}{i}\left(\frac{0{\cdot}08s}{KS}\right). \tag{9}$$

Now, if—for a reason that will be explained further on (§ 14)—the quantity between the brackets is represented by $\frac{1}{f^2}$, that is, if:[1]

$$\frac{1}{f^2} = \frac{0{\cdot}08s}{KS}, \tag{10}$$

the drift, or thrust required, may finally be written:

$$t = P\left(i + \frac{1}{f^2 i}\right). \tag{11}$$

This simple yet most important formula[2] connects the value of the thrust required for flight with the following quantities: the angle of incidence i of the equivalent flat plane, the weight P of the machine, and the quantity f, whose value depends, according to formula (10), on that of the plane area, the detrimental surface, and the lifting efficiency. The three latter are the characteristics of the aeroplane; consequently f is also a characteristic.

In accordance with the method pursued in considering the speed, it is now proposed to consider formula (11) algebraically.

The first step will be to examine the effect of any varia-

[1] By giving the coefficient K a value of 0·4, formula (10) becomes:

$$\frac{1}{f^2} = \frac{s}{5S} \tag{10a}$$

[2] According to M. Eiffel's recent researches, the equation should be more complicated, and have the form $t = aP\left(i + \frac{1}{f^2 i} + b\right)$, where a and b represent coefficients. But it has been deemed advisable in the present case to maintain the simplicity of formula (11) and subsequent formulæ, the more so since this can be done without materially affecting the general conclusions to which they give rise.

tion in the angle of incidence on the thrust, in a given aeroplane; that is, in an aeroplane with definite characteristics. The next step will be to examine the modification of this value brought about by a variation in one of the characteristics, while the angle of incidence and the remaining characteristics are kept constant.

When considering the influence of speed (and the same applies with even greater force in the present case), it was pointed out that in any procedure of this kind it is essential that any deductions that may be drawn from formulæ should always be rigorously in accord with the hypothesis under consideration; otherwise results may be reached that at first sight appear contradictory.[1]

12. Variation of thrust with the angle of incidence of a given aeroplane—Minimum thrust—Most efficient angle.

On examination it is readily seen from formula

$$t = \mathrm{P}\left(i + \frac{1}{f^2 i}\right) \tag{11}$$

that in the case of a given aeroplane, in whose case, that is, P and f are constant:

The thrust required for sustentation depends solely on the angle of incidence.

An interesting point must be considered in connection with the variation of the thrust according to the angle of incidence.

The thrust, as is known, is equal to the drift, which is composed of active resistance $\mathrm{P}i$ and head resistance $\frac{\mathrm{P}}{f^2 i}$. The former increases and diminishes with the angle of incidence, whereas the latter grows as the angle decreases

[1] A case in point was considered in § 9, in connection with the effect of an increased load on the speed of an aeroplane.

(and *vice versa*). But the product of both is constant and equal to $\frac{P^2}{f^2}$.

Now, it is admitted that the sum of two factors whose product is constant is a minimum when these two factors are equal to one another. Consequently the drift, and therefore the thrust, is a minimum when the active resistance and the head resistance are equal; and this occurs for an angle of incidence i_1, which is such that $i_1 = \frac{1}{f^2 i_1}$; that is $i_1 = \frac{1}{f}$.

This particular angle of incidence i_1, which requires the least thrust to produce the necessary lift, is called the *most efficient angle* or *optimum angle*,[1] and the minimum thrust t_1 is expressed by $2Pi_1$ or $\frac{2P}{f}$.

The flight of an aeroplane at its least angle i_1 will henceforth be called the *most efficient* or *optimum* flight, and the speed V_1 which it assumes under these conditions will be called the *most efficient* speed; that is, the speed at which the aeroplane meets with least resistance to forward motion.

13. Thrust curve.

If, in formula (11), the angle of incidence i is given various values—the characteristics of the machine remaining constant—we obtain the corresponding values of the thrust t, which may be plotted in a curve (see Fig. 12).

Just as in the case of the speed curve, the thrust curve must be confined within the limits C and D, which corre-

[1] If we considered only the planes of an aeroplane, the value of the least angle would be less that i_1, which only applies to a complete aeroplane. There therefore exists a distinction between the optimum angle of the planes and the optimum angle of the whole aeroplane. The latter angle is alone referred to.

spond to the extreme limits 0·05 and 0·20 of the angle of incidence.

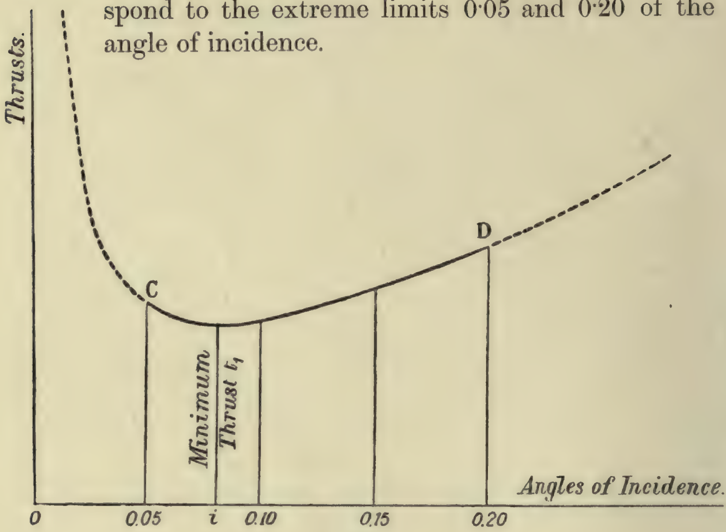

FIG. 12.

14. Fineness of an aeroplane.

Formula (11), which was set out in § 11, contained a factor f, which was of the nature $\frac{1}{f^2} = \frac{0{\cdot}08s}{KS}$, where s represents the detrimental surface of the aeroplane, S the plane area, and K its lifting efficiency. The factor f also enters into the expression of the minimum thrust t_1 which, as already stated, is equal to $\frac{2P}{f}$.

The greater the value of f, the smaller is the minimum thrust, or, what is the same thing, the minimum resistance to forward motion. Therefore, the progress through the air of the aeroplane, when its weight remains constant, becomes easier.[1]

On the other hand, the factor f also enters into the expression of the most efficient angle of incidence i_1 which

[1] The comparison here is obviously of different machines.

is equal to $\frac{1}{f}$. The greater f, the smaller the angle of incidence, so that the aeroplane driven by the minimum thrust "sails very near to the wind," like a well-designed sailing-boat.

Hence, the factor f defines a quality that may be termed the *fineness*[1] of the aeroplane, a quality that characterises the flying capabilities of an aeroplane from the point of view of least resistance.

The fineness is one of the characteristics of an aeroplane by the side of its weight, plane area, and lifting efficiency. These last two, incidentally, enter into its expression, together with the detrimental surface.

The fineness of an aeroplane becomes greater the smaller the ratio of the detrimental surface to the plane area, and the better the lifting efficiency.

As a general rule, therefore, aeroplanes with large plane area have better fineness than those of small plane area. Although any increase in the size of a plane causes an increase of the detrimental surface—since the thickness of the plane must necessarily be greater, surface friction increases, and the plane must be more strongly stayed —the ratio of the two does not remain the same, but decreases; and so the fineness of the aeroplane is improved.

Given two machines of the same plane area, that machine will have the better fineness in which the detrimental surface has been reduced to the lowest point (by the employment of stream-line structural parts, by keeping the landing chassis as small as possible, by adopting the best shape of plane, &c.).[2]

[1] *Fineness* forms perhaps an inadequate translation of the French term *finesse*, but it conveys the same sense of "delicacy," and must be used preferably to a more cumbersome expression.—*Translators.*

[2] It may be observed that an increase in the thickness of the leading edge does not increase head resistance to anything like the extent that might have been expected; but the *shape* of the forward edge seems to be of considerable importance.

Since there is a definite relation between the fineness of the aeroplane and the value of the optimum angle $\left(f=\frac{1}{i_1}\right)$, it will be convenient hereafter to define the former value by the latter, as follows:

The smaller the optimum angle, the better the fineness of an aeroplane.

Hence, formula (11) may be written:

$$t=\mathrm{P}\left(i+\frac{i_1^2}{i}\right). \qquad (11a)$$

As the result of experiments—which, by the way, are not yet complete—with several types of aeroplanes, it seems fairly well established that the Wright machines possess the greatest fineness.

For this machine the value of the optimum angle, according to M. Soreau, is about 0·06, which corresponds, for a plane area of 55 sq. m., approximately to a detrimental surface of 1 sq. m., that is $\frac{1}{55}$th of the plane area.[1]

This excellent result is due, as already stated (§ 3), to the good design of the large planes of the Wright machine, while the body and tail, which cause much friction, have been practically eliminated and the landing chassis reduced to two single skids.

The cellular-type Voisin biplane, on the other hand, has an optimum angle value of about 0·1, which corresponds to a detrimental surface of about 2·5 sq. m. for a plane area of 50 sq. m., that is, $\frac{1}{20}$th of the plane area. In this case the excessive detrimental surface should be ascribed to the tail cell, to the vertical partitions, and to the bulky chassis.[2]

[1] From formula (10) $\frac{1}{f^2}=i_1^2=\frac{0{\cdot}08s}{\mathrm{KS}}$; hence $s=\frac{\mathrm{KS}i_1^2}{0{\cdot}08}$.

In the case under consideration

$$s=\frac{0{\cdot}4\times 55\times 0{\cdot}0036}{0{\cdot}08}=0{\cdot}99.$$

[2] This type of machine is, however, gradually disappearing.

In an average aeroplane, to-day, the optimum angle value may be placed somewhere about 0·08.[1]

15. The effect of the value of an aeroplane's characteristics on the thrust.

Proceeding to analyse formula

$$(11) \qquad t = P\left(i + \frac{1}{f^2 i}\right)$$

we have now to examine the variations in thrust caused by a modification of any single one of the characteristics —the other characteristics and the angle of incidence remaining constant.

The only characteristics contained in the above formula are the weight P and the fineness f, but neither the plane area nor the lifting efficiency is included.

This çonsideration leads to a first conclusion:

If the weight and the fineness of an aeroplane are constant, the value of the thrust is not affected by that of the plane area or the lifting efficiency.

Although this conclusion may at first sight appear surprising, it will become perfectly clear if the primary hypothesis is only interpreted in an exact manner, as recommended at the end of § 9.

We have, in fact, to compare—so far as the thrust value is concerned—two aeroplanes of equal weight and equal fineness flying at the same angle of incidence. Obviously, if the plane area or the lifting efficiency is smaller in one case than in the other, the aeroplane will have to travel at a higher speed to obtain the requisite lift, so that the drift (and therefore the thrust) will be the same in either case.

Reverting once again to the analysis of formula (11), let

[1] This average value belongs principally to machines with large plane area, such as biplanes and the class of big monoplanes; while for aeroplanes of smaller area the average value is considerably higher.

C

the fineness f, and the angle of incidence i, be supposed constant, and the weight P variable.

Then the thrust is proportional to the weight of the aeroplane. Hence:

In order that two aeroplanes of different weights but of equal fineness, and flying at the same angle, may obtain the requisite lift, the thrust in either case must be proportional to the weight.

Secondly, let the weight and the angle remain constant, and only the fineness be variable, then formula (11) shows that:

The value of the thrust required to sustain an aeroplane becomes proportionately smaller as the fineness increases.

But, as a matter of fact, the above case may be subdivided into several others, since (see §§ 11 and 14) the value of the fineness depends on the plane area, the detrimental surface, and the lifting efficiency, for:

$$(10) \qquad \frac{1}{f^2}=\frac{0{\cdot}08s}{\mathrm{KS}}.$$

Now, proceeding in the same way, let two of these latter characteristics remain constant (as well as the weight and the angle of incidence), while the other is variable. Then:

The value of the thrust varies as the detrimental surface, and inversely as the plane area and the lifting efficiency.

In other words, the greater the value of the detrimental surface (the remaining characteristics being constant), the greater the necessary thrust value; similarly, the greater the plane area and the lifting efficiency, the smaller the thrust required.

This last conclusion is not in any sense in contradiction with the first conclusion set forth in the present section (p. 33), for each one refers to a different case.

Originally, the comparison was between two aeroplanes of different plane area,[1] but of the same weight, lifting efficiency, angle of incidence, and *of equal fineness,* whereas in the present case the comparison is between two aeroplanes of different plane area, but of the same weight, lifting efficiency, detrimental surface, and angle of incidence, and *of different fineness.*

Several of the foregoing results are admittedly of interest only from an academic point of view. It might be more useful to examine the effect, on the same aeroplane travelling at constant power, of the various modifications we have considered. But, as in the analogous case of the speed (see end of § 9), this would lead us outside the limits of the case under consideration. It is quite possible, too, that results found by such a method might appear to conflict with those arrived at above. This, however, would be due simply to the total difference of the nature of the two cases.

It is, in fact, quite obvious that any variation in any one of the characteristics of an aeroplane travelling at constant power would cause a modification of the angle of incidence. But the basis of the entire foregoing discussion was the supposition that the angle of incidence remained constant.

Further on (§ 31) we shall be at liberty to examine this effect of a variation in the characteristics of an aeroplane flying at constant power.

16. Value of the minimum thrust in existing aeroplanes.

The minimum value of the thrust, as previously stated (§ 12), is $2Pi_1$, where i_1 represents the optimum angle of the aeroplane. In other words:

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[1] In this case the variable characteristic is the plane area; but the same reasoning would apply if the variable characteristic was the lifting efficiency.

The minimum thrust is a fraction of the weight of the aeroplane expressed by double the value of the optimum angle.

Thus, for a Wright machine, the minimum thrust would be 0·12, or about $\frac{1}{8}$th of the weight. For a cellular Voisin biplane it would be 0·20, or $\frac{1}{5}$th of the weight.

Generally speaking, for an average machine of to-day the minimum thrust may be reckoned as 0·16—that is, about $\frac{1}{6}$th of the weight of the aeroplane.

17. Table for the rapid calculation of the thrust required to lift an aeroplane.

From the following table, which is based on formula (11), it is possible to calculate straight away the thrust required to propel an aeroplane of given fineness, flying at an angle of incidence of from 0·05 to 0·20.

Table III.

Numbers by which must be multiplied the weight of the aeroplane in order to obtain the thrust required for horizontal flight.

Angles of Incidence.	Values of the Optimum Angle denoting the Fineness.				
	0·06	0·07	0·08	0·09	0·10
0·05	0·1220	0·1480	0·1780	0·2120	0·2500
0·06	**0·1200**	0·1416	0·1666	0·1950	0·2266
0·07	0·1214	**0·1400**	0·1614	0·1857	0·2128
0·08	0·1250	0·1412	**0·1600**	0·1812	0·2050
0·09	**0·1300**	0·1444	0·1611	**0·1800**	0·2011
0·10	0·1360	0·1490	0·1640	0·1810	**0·2000**
0·11	0·1420	0·1545	0·1681	0·1836	0·2009
0·12	0·1500	0·1608	0·1733	0·1875	0·2033
0·13	0·1577	0·1677	0·1792	0·1923	0·2069
0·14	0·1657	0·1750	0·1857	0·1978	0·2114
0·15	0 1740	0·1827	0·1927	0·2040	0·2167
0·16	0·1825	0·1906	0·2000	0·2106	0·2225
0·17	0·1911	0·1988	0·2076	0·2176	0·2288
0·18	0·2000	0·2072	0·2155	0·2250	0·2355
0·19	0·2089	0·2158	0·2237	0·2326	0·2426
0·20	0·2181	0·2245	0·2320	0·2405	0·2500

The fineness is given by the value of the equivalent optimum angle, between 0·06 and 0·10.

In order to obtain the desired result, multiply the weight of the aeroplane by the number situated at the intersection of the line and the column corresponding respectively to the values of the angle of incidence and the fineness (optimum angle).

Throughout the present work it should be remembered, all numerical calculations are approximate only; results thus obtained must be regarded simply as an indication of the truth.

Example—

Calculate the thrust required for an aeroplane weighing 480 kg., whose fineness is denoted by an optimum angle 0·07, flying at an angle of incidence 0·13.

The number in the above table situated at the intersection of line 0·13 and column 0·07 is 0·1677.

Multiplying the weight, 480 kg., of the aeroplane by this number gives the thrust required for horizontal flight, *i.e.* 80·498 kg., or roughly 81 kg.

IV.—Power

18. Useful power required for sustentation.

When, through the effect of a thrust t (expressed in kilogrammes), an aeroplane flies horizontally through a space V (expressed in metres), the work required to produce this result can be represented by the product Vt (in kilogrammetres).

If the space in question is covered in one second, V, which is its length in metres, is also the speed of the aeroplane in metres per second.

The product Vt then represents, in kilogrammetre-seconds, the mechanical power required to enable the aeroplane to remain in horizontal flight.

This power is obviously *useful power.* Let it be represented by the symbol T_u; then, adopting the usual unit of power, the horse-power (1 H.P. = 75 kilogrammetre-seconds), we can write:

$$T_u = \frac{Vt}{75}.$$

If in the above equation the symbols V and t are replaced by the values obtained from formulæ (5) and (11), we obtain the expression of the useful power required to propel an aeroplane in terms of the characteristics of the aeroplane and of its angle of incidence:

$$(12) \qquad T_u = \frac{1}{75\sqrt{K}} \cdot P\sqrt{\frac{P}{S}} \cdot \left(\sqrt{i} + \frac{1}{f^2 i\sqrt{i}}\right).$$

19. Motive power required for sustentation.

In order to obtain the useful power required for sustentation from the propelling plant (motor and propeller), the motor must develop a greater *motive power.* The relation between the useful power and the motive power gives the *efficiency* of the propelling plant.

It will be shown below (§§ 29 and 87) that the efficiency varies with the relation of the speed of revolution of the propeller to the travelling speed of the aeroplane, and reaches a maximum for a certain value of this relation.

Propellers are built at the present day that give a maximum efficiency of 75 per cent., a figure which will no doubt be exceeded in future. But every propeller is by no means so efficient, nor are propellers always run under the most favourable conditions; consequently, it would be unwise to credit the average propelling system of the present day with a higher efficiency than 50 per cent.

This being so, the motive power required for horizontal flight is:

$$(13) \qquad T_m = 2T_u.$$

Pursuing the same method as in the case of the value of the speed and the thrust, we may now proceed to consider algebraically the formula giving the power value. Formula (12), expressing the useful power, will be first considered; the results thus obtained will next (§ 23) be brought to bear on the motive power.

Firstly, we can proceed to examine the effect of a variation of the angle of incidence on the value of the useful power required to propel a given aeroplane.

Next, we may consider the manner in which this value is affected by a modification of one of the characteristics, the remaining characteristics and the angle of incidence remaining constant.

20. Variation of the useful power with the angle of incidence in a given aeroplane—Minimum power—Economic angle and speed.

On examination, it is clear from formula (12) that in the case of a given aeroplane—that is, one in which the values of P, S, K, and f are constant:

The value of the useful power required for flight depends simply on the angle of incidence.

As in the case of the thrust, so too the value of the useful power is a minimum for an angle of incidence of a given value; this angle, however, is not the optimum angle i_1, but the product of this angle and $\sqrt{3}$ or 1·732. For it can be shown that the expression $\sqrt{i}+\frac{1}{f^2 i\sqrt{i}}$, or its equivalent $\sqrt{i}+\frac{i_1^2}{i\sqrt{i}}$, is a minimum for a value of i equal to $i_1\sqrt{3}$.[1]

Consequently, this latter angle of incidence requires

[1] It is clear that, for this value of the angle, the value of the active resistance Pi is three times greater than that of the passive resistance $\frac{Pi_1^2}{i}$.

the least expenditure of power for flight, and is therefore the most economical. Hence it may be termed the *economical angle*, and may be expressed:

$$i_e = i_1 \sqrt{3}.$$

The speed at which the aeroplane travels at this angle may be called the *economical speed*, and horizontal flight at least power, *economical flight*.

In a Wright aeroplane the value of the economical angle is roughly: $0{\cdot}06 \times 1{\cdot}732 = 0{\cdot}11$. For a cellular Voisin biplane the angle is: $0{\cdot}1 \times 1{\cdot}732 = 0{\cdot}17$ about. For an average present-day machine, finally, it is: $0{\cdot}08 \times 1{\cdot}732 =$ about $0{\cdot}14$.

21. Curve of useful power.

In Fig. 13 are reproduced the curves, that have already been found, of the speed AB (Fig. 9), and of the thrust CD (Fig. 12), of a given aeroplane. On each perpendicular MX, defined by a certain value OM of the angle of incidence, is marked off a length MQ proportional to the product of the corresponding values of the speed and the thrust (measured respectively by the lengths MN and MP). The resulting curve EF represents the variation, with the angle of incidence, of the useful power required to sustain an aeroplane (for this, as already known, is expressed by the product Vt).

By thus placing the three curves side by side, it is clearly seen why the minimum thrust and the minimum power do not correspond to the same angle of incidence.

For, as the angle of incidence increases from a low value (from the lowest possible value $0{\cdot}05$) to the value i_1 of the optimum angle, both the speed and the thrust diminish. Consequently, their product—which is equal to the useful power—must necessarily diminish likewise, as is clearly shown by Fig. 13.

As the angle increases beyond i_1 the speed continues

to diminish, although more slowly (see § 8), but the thrust begins to increase. But as the increase in the thrust is only very slight at first, the value of the speed continues preponderant in the product, and the power still continues

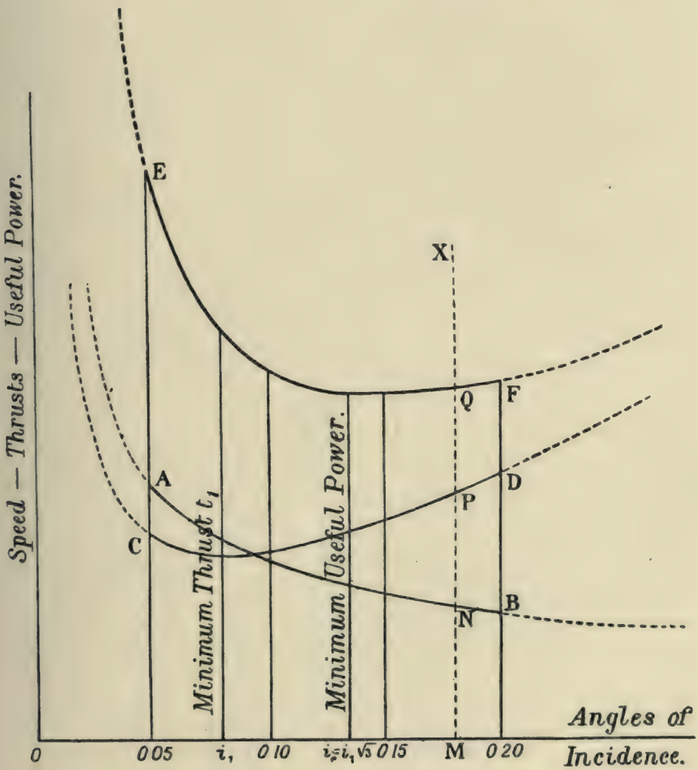

FIG. 13.

to diminish; the rate of this decrease, however, gradually becomes slower, until, at a certain value i_e of the angle, the increase in the thrust exactly balances the decrease in the speed. The power therefore soon ceases to diminish and starts to increase when the angle of incidence passes beyond the value i_e. Consequently the minimum thrust

is found at this angle, which, as already seen, is equal to $i_1\sqrt{3}$.

The curve clearly shows the rapid increase of the useful power as the angle of the planes diminishes. Hence:

The high speeds attainable with very small angles require an enormous expenditure of mechanical power.

The above consideration, together with the danger of endeavouring to "sail too close to the wind," rigidly limits the use of very small angles of incidence.

22. Effect of the value of the characteristics on the useful power.

Proceeding with the discussion in due order of formula

$$(12) \qquad T_u = \frac{1}{75\sqrt{K}} \cdot P\sqrt{\frac{P}{S}} \cdot \left(\sqrt{i} + \frac{1}{f^2 i\sqrt{i}}\right)$$

brings up the questions of the variations in the useful power due to a modification of one of the aeroplane's characteristics, the remaining characteristics and the angle of incidence being constant.

The formula contains all four characteristics: the weight P, the plane area S, the lifting efficiency K, and the fineness f. Each one of these characteristics will in turn be assumed to be variable, while the other three and the angle remain constant.

In the first place, it should be noticed that the useful power is proportional to the product $P\sqrt{\frac{P}{S}}$, that is, of the weight multiplied by the square root of the loading. Hence, if the weight is variable:

An increase in the weight causes an increase in the power proportional to the product of the ratio of increase and the square root of this ratio. And inversely.

If the weight, for instance, is increased four-fold (the other characteristics and the angle remaining constant)

the useful power required for flight grows eight-fold (though the speed is only doubled).

Secondly, if the plane area S is variable, the useful power is inversely proportional to $\sqrt{S}$. Hence:

A decrease in the plane area causes an increase in power proportional to the square root of the ratio of decrease. And inversely.

Thus, if the plane area of an aeroplane is reduced to one-quarter, other things being equal, the power required for flight is doubled (and so is the speed).

High-speed aeroplanes that may be built in the future will probably be designed with small plane area [1] rather than of heavy weight, since a similar increase of speed requires in the former case only an equal increase of power, whereas in the latter case it would require an increase of power proportional to the cube of the increase in speed.

Thirdly, if the lifting efficiency K is variable, the useful power is inversely proportional to $\sqrt{K}$. Hence:

The better the lifting efficiency, the smaller the power required for flight.

Thus the advantage of speed to be derived by diminishing the lifting efficiency is, as already stated (§ 9), wholly illusory.[2] At the same time, it is certainly possible (§§ 6 and 9) that in the future the lifting efficiency may be purposely diminished in flight to accelerate the rate of travel.[3] But this would require a sacrifice of power;

[1] A tendency which is already clearly discernible.

[2] This is seen even more clearly by examining the relation existing between the useful power and the speed, which, as stated further on (§ 26), is:

$$T_u = \frac{1}{75}\left(\frac{P^2}{KSV} + 0{\cdot}08 s V^3\right).$$

This shows that when the speed V is assumed to be constant the useful power required for flight is smaller, the greater the lifting efficiency K.

[3] Certain constructors already, in fact, tend to return to the use of flat planes for the wings in order to gain speed. But they are forced to employ excessive power.

which, however, is admissible, since in normal flight there is usually an excess margin of power available (see §§ 25 and 30).

And lastly, if the fineness f is variable:

An increase in the fineness decreases the power required for flight.

Thus, as was foreseen, the aeroplane with the greatest fineness, other things being equal, requires the least power for flight.

The distinctions set forth in considering the same steps in the discussion of the thrust are not necessary here, for the assumption that the characteristics other than the fineness are constant presupposes that the fineness can only vary with the detrimental surface s.

23. Application to the motive power of the results concerning the useful power.

It has already been shown (§ 19) that the ratio of the useful power to the motive power actually expended gives the measure of the efficiency of the propelling plant. Hence, one can calculate the motive power required for flight by dividing the useful power by the efficiency of the propelling plant. The useful power is given by formula (12); but this formula contains one factor, the angle of incidence, which affects the speed of the aeroplane and, consequently, the efficiency of its propelling plant (§ 19), since this efficiency varies with the ratio of the speed to the rate of revolution of the propeller. Thus the above method of calculating the motive power must only be applied if the propelling efficiency and the angle of incidence are given their suitable values.

The same is true regarding the application of the results obtained from the discussion of formula (12) to the variations of the motive powers.

Such an application is only possible if the propelling efficiency is assumed to be constant; as, for instance, in

the case where a comparison is made between two aeroplanes differing from one another by one of their characteristics only, the angle of incidence being constant. In such a case one may assume the efficiency of the two propelling plants to be identical.

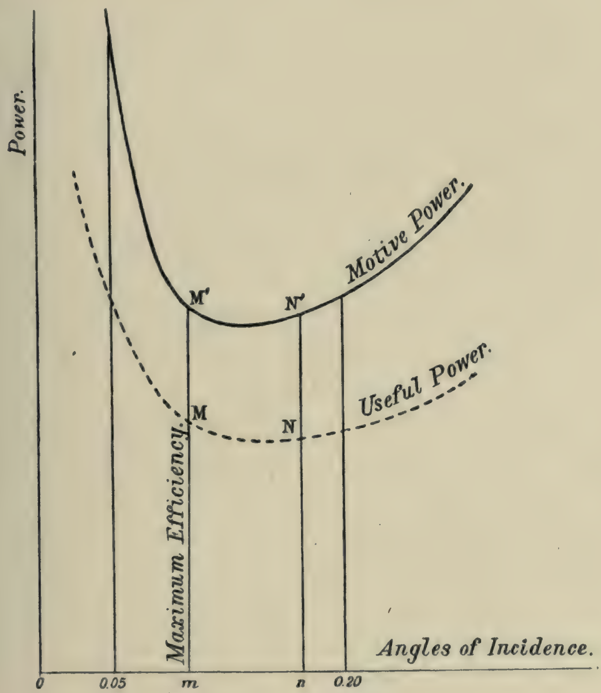

FIG. 14.

On the other hand, the propelling efficiency can no longer be held constant in considering the variation of the motive power required for flight in *one and the same machine.* Consequently, the curve giving the variations of the useful power with the angle of incidence shown in Fig. 13, cannot be applied to the variations of the motive

power, simply by increasing the length of the verticals in inverse proportion to a fixed efficiency.

Such variations of the motive power can be indicated in a curve of the form shown (unbroken line) in Fig. 14.

For any value On of the angle of incidence, the ratio $\frac{Nn}{N'n}$ gives the propelling efficiency. This ratio varies according to the position of the perpendicular N'Nn and attains a maximum when the latter occupies a position M'Mm, which corresponds to that value of the angle at which the aeroplane travels at the speed best suited to the rate of revolution of its propeller.

24. Tables for rapidly calculating the power required to fly an aeroplane.

As previously seen (§ 18), the useful power, in horse-power, required for flight $= \frac{Vt}{75}$. Also, it has been shown (§ 19) that the motive power is equal to twice the useful power, that is, to $\frac{Vt}{37 \cdot 5}$, when the efficiency had its average value of 50 per cent.

From Tables I., II., and III. (§§ 10 and 17), which serve to calculate the speed and the thrust from the characteristics of the aeroplane and from its angle of incidence, it is therefore also possible to calculate the useful or motive power required for flight, irrespective of the propelling efficiency.

Example:—**Calculate the motive power required for horizontal flight, at an angle of 0·13, of an aeroplane weighing 480 kg., with a plane area of 40 sq. m.; the fineness being $\frac{1}{0 \cdot 07}$ (corresponding to an optimum angle 0·07) and with a propelling efficiency of 50 per cent.**

The examples in §§ 10 and 17 related to a similar aeroplane, and the values then found for the speed and thrust respectively were 15·19 m.p.s. and 80·496 kg.

The product Vt will therefore be approximately 1222·7. The fraction $\frac{1}{37 \cdot 5}$ therefore—that is, about 32·6—represents the necessary motive power in horse-power. Again, $\frac{1}{75}$th of the same value — that is, 16·3 H.P. — represents the useful power required for flight.

For any other efficiency, all that is required to obtain the motive power is to divide the useful power (16·3) by the efficiency in question.

From Table IV., in conjunction with Table II., which is reprinted once again, it is possible to calculate directly the motive power required for horizontal flight, at various angles of incidence, of aeroplanes of different fineness (defined by optimum angles of from 0·06 to 0·1), assuming only that the lifting efficiency K is 0·4 and the propelling efficiency 50 per cent.

TABLE IV.

For the calculation of the motive power required for horizontal flight. (Lifting efficiency = 0·4 ; propelling efficiency = 50 %.)

Angles of Incidence.	Value of the Optimum Angle defining the Fineness.				
	0·06	0·07	0·08	0·09	0·10
0·05	0·07263	0·08811	0·10595	0·12620	0·14884
0·06	0·06531	0·07707	0·09068	0·10612	0·12332
0·07	0·06111	0·07049	0·08125	0·09348	0·10713
0·08	0·05890	0·06653	0·07539	0·08539	0·09661
0·09	0·05777	0·06418	0·07160	0·08000	0·08937
0·10	0·05739	0·06288	0·06920	0·07638	0·08439
0·11	0·05703	0·06205	0·06752	0·07374	0·08069
0·12	0·05781	0·06197	0·06679	0·07226	0·07834
0·13	0·05824	0·06193	0·06618	0·07102	0·07642
0·14	0·05908	0·06239	0·06620	0·07052	0·07537
0·15	0·05994	0·06294	0·06639	0·07028	0·07466
0·16	0·06084	0·06354	0·06666	0·07021	0·07417
0·17	0·06185	0·06433	0·06717	0·07042	0·07405
0·18	0·06289	0·06516	0·06777	0·07076	0·07406
0·19	0·06389	0·06601	0·06842	0·07114	0·07420
0·20	0·06506	0·06696	0·06920	0·07173	0·07458

In order to find the required power, multiply the weight of the aeroplane by the number in Table IV. at the intersection of the line and the column corresponding respectively to the value of the angle of incidence and the fineness (optimum angle), and multiply the product by the number in Table II. corresponding to the loading.

TABLE II. (reprinted)

Loading in kg. per sq. m. . .	4	5	6	7	8	9	10	11	12
Multipliers . . .	0·632	0·707	0·775	0·837	0·894	0·949	1·000	1·049	1·095
Loading in kg. per sq. m. . .	13	14	15	16	17	18	19	20	21
Multipliers . . .	1·140	1·183	1·225	1·265	1·304	1·342	1·378	1·414	1·449
Loading in kg. per sq. m. . .	22	23	24	25	26	27	28	29	30
Multipliers . . .	1·483	1·517	1·549	1·581	1·612	1·643	1·673	1·703	1·732

So far as the accuracy of these results is concerned, warnings previously given in similar cases must be taken into account.

If the propelling efficiency is other than 50 per cent., the results obtained only require to be divided by the ratio of the efficiency to 50 per cent.

Example of how to use these tables

Solve the problem set forth at the beginning of the present paragraph.

The number in Table IV. at the intersection of line 0·13 and column 0·07 is 0·06193. Since the loading is $\frac{480}{40} = 12$ kg. per sq. m., the corresponding multiplicator in Table II. is 1·095. The motive power required is therefore:

$$480 \times 0{\cdot}06193 \times 1{\cdot}095,$$

that is, about 32·6 H.P., as previously found.

V.—Adaptation of the Propelling Plant to the Aeroplane [1]

25. Preliminary considerations.

It has already been stated (§§ 20 and 21) that the useful power required to maintain horizontal flight depends on the angle of incidence and increases as the latter diminishes.[2]

Again, it is known that a decrease in the angle brings about an increase in the speed of the aeroplane; and, consequently, for the attainment of higher speeds greater motive power is required.

Hence a given aeroplane could be flown at as high a speed as desired provided it was furnished with a sufficiently powerful motor, if it were not for the fact that the danger of diminishing the angle places a limit to the increase in speed. Therefore:

The greatest speed attainable by an aeroplane is that beyond which the diminution of the angle of incidence becomes dangerous.

The value of this *critical speed*, therefore, depends on the structure of the aeroplane itself, on its capacity of flying at low angles, on the value of its characteristics; **but it is independent of engine power.**

In order to attain this critical speed, all that is required is to provide the aeroplane with a propelling plant capable of producing the useful power necessary for flight at the angle of incidence corresponding to the critical speed in question. But, as a rule, aeroplane designers, in order to

[1] Those readers who wish to gain, without going too deeply into every question, a complete survey of the problem of the aeroplane, are recommended to skip this chapter for the moment, or at all events only to examine its conclusions.

[2] This is correct only if the angle of incidence is less than the economical angle, as may be seen from the curve in Fig. 13.

facilitate starting and climbing,[1] prefer to provide their machines with motors considerably more powerful than would be required simply to maintain horizontal flight at the critical speed.

So as not to go beyond this speed the motor must be prevented from running all out by regulating the admission of the gas. Therefore flight, with the motor throttled down, is the general rule in practice.

Nevertheless, it is essential to examine the case where the motor develops its full power, and is running all out; for this will provide important information regarding the most efficient utilisation of a given propelling plant to sustain a given aeroplane, that is, *the application of the propelling plant to the aeroplane.*

The method hereafter followed to solve this question is very simple, although it may prove somewhat lengthy. It consists in setting out by means of curves, firstly, the law of variation of the useful power required to propel an aeroplane with the speed of flight; secondly, the law of variation of the useful power given by the propelling plant (the motor—working at full power—and the propeller) with the speed of flight of the aeroplane to which this propelling plant is adapted; and, lastly, to approximate these two curves.

This method of research will also lead to some interesting conclusions regarding the manner in which a variation of the characteristics affects the conditions of horizontal flight of a given aeroplane driven by a given propelling plant; in the previous discussions this very question, as will be remembered, was purposely left aside.

Finally, some use will be made of the curves to be established from the consideration of ascending flight, that is, climbing.

[1] And also for reasons connected with the nature itself of the petrol engine, which comprises a number of cylinders ; wherefore, it is necessary to provide sufficient power for horizonal flight even if one or more cylinders stop working.

26. Curve representing the variation of the useful power required to sustain an aeroplane as a function of its speed.

Reference has already been made (§ 22, footnote, p. 43) to the relation connecting the useful power required for

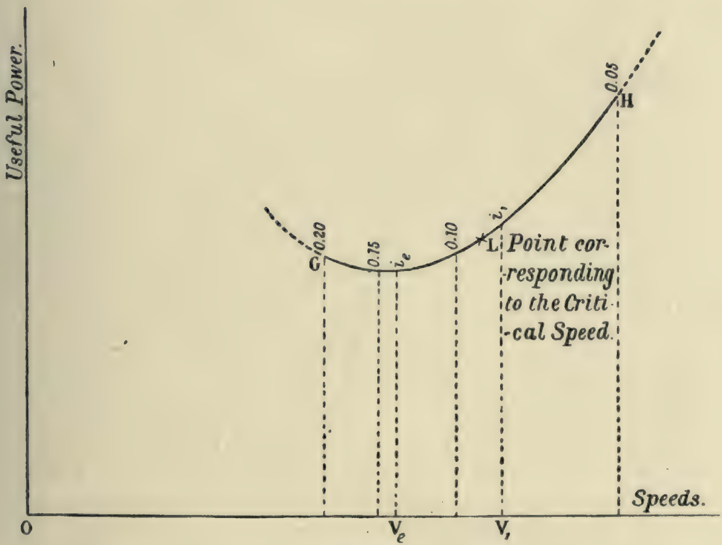

FIG 15.

flight to the horizontal flight-speed of an aeroplane. The relation may be established in the following manner :—

The thrust required to sustain an aeroplane is, from formula (8) (§ 11), $Pi + 0{\cdot}08sV^2$, where P, i, s, and V stand respectively for the weight, the angle of incidence, the detrimental surface, and the speed.

By substituting for i its equivalent $\frac{P}{KSV^2}$ obtained from the fundamental formula (4), we obtain :

$$(14) \qquad t = \frac{P^2}{KSV^2} + 0{\cdot}08sV^2,$$

the equation connecting the thrust required to sustain an aeroplane with the speed assumed in flight.

Next, by taking the expression $\frac{Vt}{75}$ of the useful power, and substituting therein for t the values obtained from the above equation, we finally obtain the relation required:

$$T_u = \frac{1}{75}\left(\frac{P^2}{KSV} + 0{\cdot}08sV^3\right), \tag{15}$$

or, introducing the factor of fineness:

$$T_u = \frac{1}{75}\left(\frac{P^2}{KSV} + \frac{KSV^3}{f^2}\right). \tag{15a}$$

This relation can be diagrammatically shown in the curves in Fig. 15.

If this curve is completed by the insertion, at various points, of the corresponding values of the angle of incidence, it will contain in a single graph all the data relating to the variation of the useful power required to sustain a given aeroplane.

This graph may be termed *the characteristic curve,* and constitutes the first of the two curves referred to in § 25.

27. Normal speed.

One particular point in this curve is of special interest; this is the point L, which corresponds to the critical speed mentioned in § 25, that cannot be exceeded without danger.

Obviously, it would be advantageous to fly at this speed, provided the motor is sufficiently powerful; in which case the critical speed becomes the *normal speed.*

On the other hand, if the propeller is incapable of developing the useful power required for sustentation—even though the motor be running at full power—the normal flight of the aeroplane takes place at a lower speed. As a general rule, however, the motor should—at full power—be powerful enough to enable the critical

speed to be attained; and it may be further pointed out (see § 25) that the pilot should have a reserve of power enabling him to exceed the critical speed.

Usually, the normal speed of an aeroplane lies between its highest speed V_1 and its economical speed V_e. As a matter of fact, the more closely the normal speed approaches the speed V_1 the better; for it is not difficult to see that, on comparing the optimum rate of travel and the economical rate of travel, the former gives an increase of speed of 32 per cent., and only requires an increase of power of 13 per cent.[1]

28. Variation of the power developed by an explosion motor according to its rate of revolution.

The power developed by a petrol motor, such as used in aviation, depends on the number of its revolutions (when running at full power). To render this point clearer, it may be explained that if, for the purpose of increasing the power of a motor, a Prony brake[2] is connected to its shaft, the motor will assume a certain angular speed equal to n_1 revolutions per second. The power measured is equal to T_{n_1}.

If the tension of the brake is modified, the motor will run at a new normal speed n_2, and the power will become

[1] The values of the speeds V_1 and V_e are respectively $\sqrt{\frac{P}{KSi_1}}$ and $\sqrt{\frac{P}{KSi_1\sqrt{3}}}$; so that their ratio is $\sqrt{\sqrt{3}}$, or $\sqrt{1{\cdot}73}$., that is about 1·32. The values of the powers T_1 and T_e are respectively V_1t_1 (that is, $2Pi_1V_1$) and V_et_e, that is, $Pi_1\left(\sqrt{3}+\frac{1}{\sqrt{3}}\right)V_e$; their ratio is therefore equal to $\frac{V_1}{V_e}\times\frac{2}{\sqrt{3}+\frac{1}{\sqrt{3}}}$, or $1{\cdot}32\times0{\cdot}86$, that is 1·13. Hence it will be seen that the optimum speed V_1 is greater by 32 per cent. than the economical speed V_e, whereas T_1 is greater than T_e by only 13 per cent. of the latter's value.

[2] The use of a dynamometer is more generally adopted for testing the power of a petrol engine.

T_{n_2}. By making a series of similar tests the power of the motor may be ascertained for its various speeds of revolution. The power curve will have the shape shown in Fig. 16.

It will be seen that the motor develops its highest power at a certain speed N_1.[1] To obtain the best results from any engine it should therefore always be run at this speed, which is always indicated by its constructor.

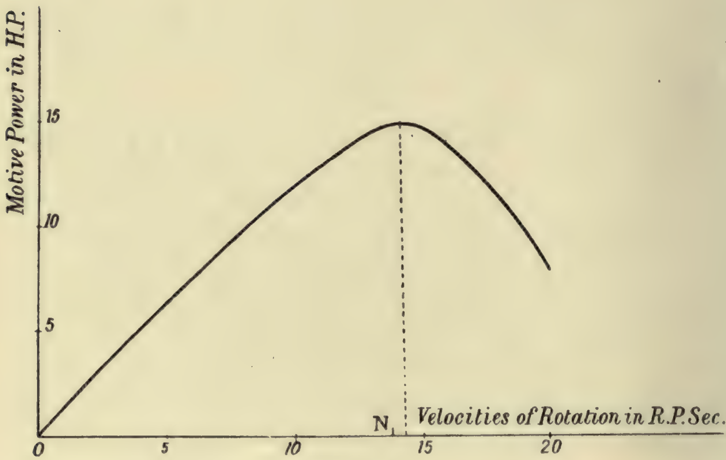

FIG. 16.

It will, of course, be evident that at any given speed n it is possible by throttling down the engine to make it develop only a fraction of the power it is shown to develop at its normal speed in Fig. 16. As a matter of fact, the power developed by an engine running throttled down to various degrees could be represented by a series of curves similar to the one in Fig. 16, but of increasing flatness the greater the throttling effect.

[1] It is also worthy of note that the rising part of the curve is practically a straight line. Hence the power developed by a motor, up to the point N_1, where it develops its maximum power, may be considered as approximately proportional to its number of revolutions.

29. Curve of the useful power furnished by a propelling plant at different speeds—Motive power expended—Maximum efficiency of a propelling plant.

If a screw-propeller is mounted on the motor shaft and the motor set to run at full power, a normal speed of rotation, n_o, will be attained, at which the power absorbed by the propeller in beating the air is exactly the same as that developed by the motor at this speed of revolution; in Fig. 16 this power, T_o, would correspond with the number of revolutions per second n_o. The propeller exerts a *thrust*, J_o, on the motor, but of course without moving it from its position, since it is supposed to be fixed.

But when the motor is mounted on an aeroplane, the thrust of the propeller gives it a forward speed, V_1, whose value depends on the air resistance encountered by the aeroplane in its forward motion.

There is thus set up a normal speed of rotation, n_1, which is such that the power absorbed by the propeller is equal to the power, T_1, put out by the motor. As a matter of fact, the number of revolutions per second, n_1, is slightly greater than the number of revolutions n_o, which corresponded in the first case to the rotation of the propelling plant. On the other hand, the thrust J_1 is weaker than the thrust J_o, which was produced when the apparatus was stationary.

When the propeller was working in a stationary position it was only called upon to produce the thrust J_o, but no actual *work* was produced, since the point to which the thrust was applied remained stationary.

On the other hand, when the thrust J_1, created by the rotation of the propeller, causes the apparatus to move forward freely *useful work* is produced, since the function of the propeller is to cause the aeroplane to move forward.

The propelling plant therefore, by producing *useful work*, which can be expressed in kilogrammetres by V_1J_1

and in H.P. by $\frac{V_1 J_1}{75}$, puts out again a certain amount of the power T_1 developed by the motor. The *efficiency* of the entire propelling plant can therefore be expressed by $\frac{V_1 J_1}{75 T_1}$ (the power T_1 being assumed as expressed in H.P.).

If the aeroplane is rearranged so that its head resistance is reduced, each value of this head resistance will have a corresponding value of the speed of flight V, the number of revolutions per second n, the motive power T, the useful power $\frac{VJ}{75}$, and the efficiency $\frac{VJ}{75T}$.

By approximating the values of the motive power and of the useful power to that of the speed of flight, a diagram may be drawn which expresses the two former as a function of the latter. This diagram is shown in Fig. 17.

It includes two curves; the former, TKT, representing the variation of the motive power, T, developed, and the speed of flight V; the second showing the variation with the same speed of the useful power $\frac{VJ}{75}$ put out by the propelling plant.

To each point N on the first curve corresponds a point, N′, on the second; if the speed of flight is On, the efficiency of the propelling plant is given by the ratio $\frac{N'n}{Nn}$, of the useful power produced to the motive power developed to produce it. Further, to each couple of points N′N corresponds a definite speed of rotation, which may be entered on the diagram to complete it.

The highest point L′ of the curve OL′U corresponds to a certain value $Ol = V_L$ of the speed of flight of the aeroplane, the number of revolutions then being n_L. Fig. 17 shows, however, that as a rule the efficiency of the propelling plant is not a maximum at this particular speed

of rotation, but at the speed of rotation $Om = V_M$, which is such that the tangents to the two curves at M and M′ are parallel. The number of revolutions per second at which the propeller then rotates is n_M.

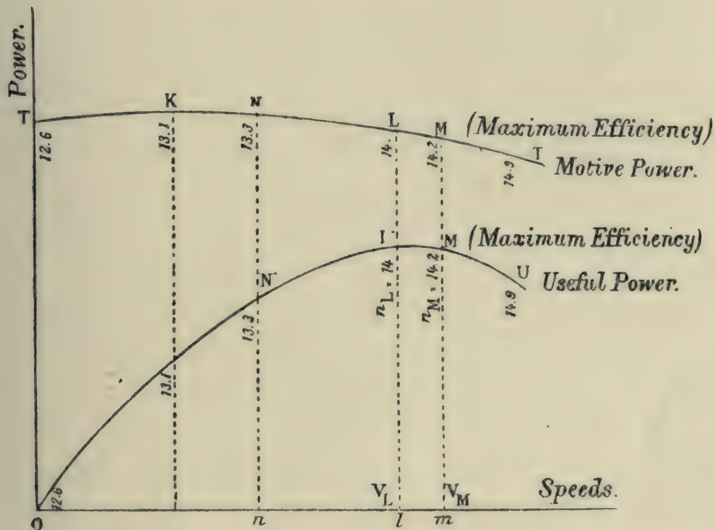

FIG. 17.—The vertical numbers indicate the speeds of rotation of the propelling plant.

Thus, as has been previously stated (§ 19)—and the conclusion is of the highest importance:

A given propeller does its maximum work under certain conditions, which are defined by a definite relation between the speed of revolution of the propeller, and the speed of flight of the aeroplane it propels.

The curve TKT in the diagram of powers also has a highest point K. But in the general case illustrated in Fig. 17, that is with an undefined motor and propeller, this point is not situated vertically above the highest point L′ in the useful-power curve, nor above M′, which indicates the highest efficiency. The practical meaning

of this lack of correspondence is that when any propeller is affixed to any motor, the joint system produces respectively its maximum useful power, and its maximum efficiency at speeds of rotation that differ from the speed at which the motor must rotate to develop its maximum power (§ 28).

It is obviously desirable that these various speeds should coincide, so that the whole of the power developed by the motor may be utilised when the whole system is rotating at the propeller's most suitable velocity.

In principle,[1] therefore, the maximum power of the motor, and the maximum efficiency of the whole propelling plant should be developed at the same speed (of rotation and flight speed of the aeroplane). This condition is represented by a relation between the power of the motor and its speed of rotation, and the coefficients of this relation depend on the dimensions and the shape of the propeller.[2]

If, therefore, a motor of given horse-power is to drive a given propeller, this motor must in principle develop its maximum power at a certain definite velocity of rotation and at no other.

When the above condition is realised, the power-velocities diagram assumes the form shown in Fig. 18.

It should be noticed that in this case the upper curve TMT is very much flattened out. It may therefore be laid down, more especially if the flight speed must be

[1] In some cases it may be useful (see § 30) to make the motor rotate at a slightly higher normal velocity than the velocity giving the maximum power.

[2] For screw-propellers of the usual type (see § 87) this relation may be fixed approximately and *merely by way of an indication* as $T_m = 0{\cdot}0001\, n^3 D^5$, where T_m stands for the motive power in H.P., n the velocity of rotation in rev. per sec., and D the diameter of the propeller in metres. *Example:* —If a motor of 16 H.P. actuates an average propeller of two metres diameter, the motor should develop its 16 H.P. at a velocity of rotation n so that $n^3 = \frac{16}{0{\cdot}0001 \times 32}$; that is, at 17·1 rev. per sec. or 1025 r.p.m.

confined within narrow limits—which is usually the case with the aeroplane—*that the motive power remains practically constant.*

Finally, it may be noticed that the velocity of rotation of the propelling plant varies but slightly, as the flight speed of the aeroplane runs through the usual values from

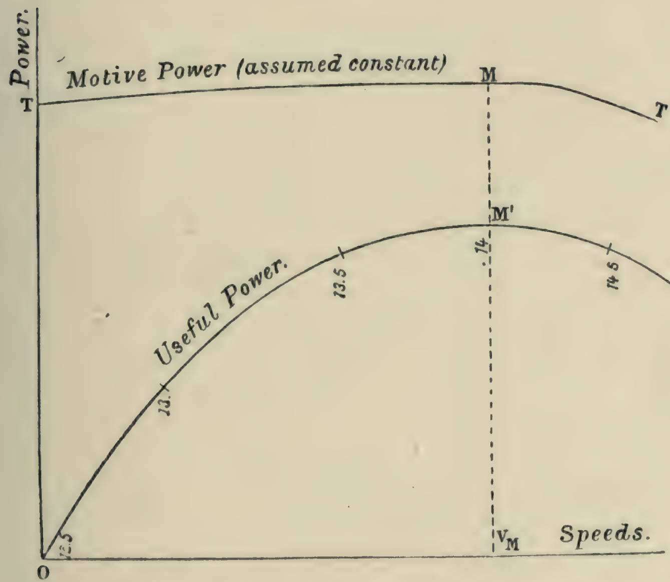

FIG. 18.—The vertical numbers indicate the velocities of rotation of the propelling plant.

zero upwards. So long as this flight speed remains within fairly narrow limits, the variation in the number of revolutions of the propeller is only slight. This explains the reason for the flatness of the motive power curve in Figs. 17 and 18; and, as a matter of fact, this curve would become a horizontal straight line if the velocity of rotation and, consequently, the motive power remained exactly constant.

To sum up: the main point to be remembered is that

the working of a propelling plant, consisting of a motor and a propeller suited to one another, has the character, at different flight speeds, of the curve OM′U (Fig. 18), showing the useful power given out by the propeller at its various velocities.

This diagram, which is the second referred to in § 25, will hereafter be called *the characteristic curve of the propelling plant.*

In order to complete it, the values corresponding to the angular velocity n may be added, and the graph will then include every point relating to the working of the propelling plant. *The motive power remains practically constant,* as already stated.

It may here be added once again, that the foregoing remarks apply only to an engine running at full power. If the motor is throttled down, the working of the propelling plant could be shown, according to the extent of throttling, by a series of curves similar to curve OM′U in Fig. 18; but these curves would grow flatter, the greater the throttling.

30. Approximation of the aeroplane and propelling plant diagrams—Deductions.

Interesting results are obtained by instituting a close approximation between the two diagrams. This may be done by drawing in the same (Fig. 19) the aeroplane curve GH, and the propelling-plant curve IJ (the motor running at full power). Only those portions of the curves need be drawn that correspond to the attainable speeds (see § 8).

Firstly, in order that horizontal flight may be possible, GH must not lie wholly above curve IJ (Fig. 20); for if it did, it would be impossible to find a speed such that the useful power given out by the propeller was exactly that required to sustain the aeroplane. In such a case the aeroplane would be unable to maintain horizontal flight,

and the most its propelling plant could effect would be to prolong the glide.

When the two curves are tangential (Fig. 21), horizontal flight is possible only at the single speed V_R corresponding to the point of contact R.

The aeroplane would be unable to rise from the ground, for this requires (as will be shown in § 44) more useful power than is needed for sustentation.

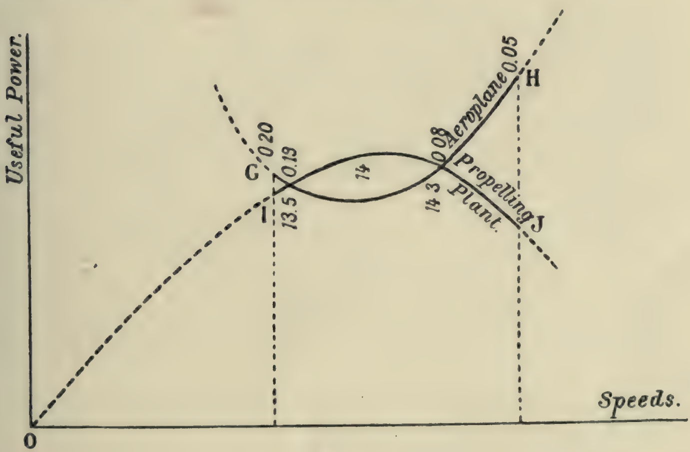

FIG. 19.

To render horizontal flight feasible, the two curves must intersect (Fig. 22). This they do at two points, A_1 and A_2.

There are thus two values, Oa_1 and Oa_2, of the speed at which the propeller furnishes the requisite useful power to sustain the aeroplane (when the motor is working at full power). The horizontal flight of the aeroplane will therefore be made at one of these two speeds; that one being the greater, Oa_2, since there is an advantage from every point of view in choosing the higher speed.[1] Further, this

[1] Several considerations that will be set out further on (§ 44), in any case, make the choice of this speed practically compulsory.

speed Oa_2 must be inferior to the critical speed Ol of the aeroplane, to which the point L corresponds in curve GH, otherwise it would be dangerous to run the motor at full power in horizontal flight.

By a suitable modification of the angle of incidence the aeroplane may be flown at any speed between Oa_1 and Oa_2;

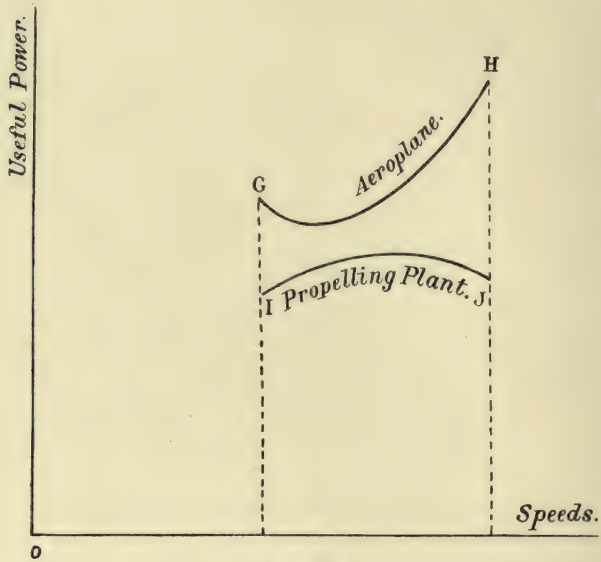

FIG. 20.

for instance, at the speed On corresponding to the points N and N′ on the curves.

But in this case *it is essential that the motor should not be run at full power, but should be throttled down.*

At the speed On, in fact, the useful power $N'n$ given out by the propeller is greater than the useful power Nn required to sustain the aeroplane, which consequently tends to ascend, so that, in order to maintain the flight-path horizontal, the motor must be throttled down.

By operating the elevator it is therefore possible to

make the propelling plant (*the motor running at full power*) *furnish an excess of useful power* which may be turned to use in various ways, as will be shown hereafter (§§ 44 and 45).

This excess of useful power reaches a maximum when the aeroplane is travelling at a certain speed, Om (that is,

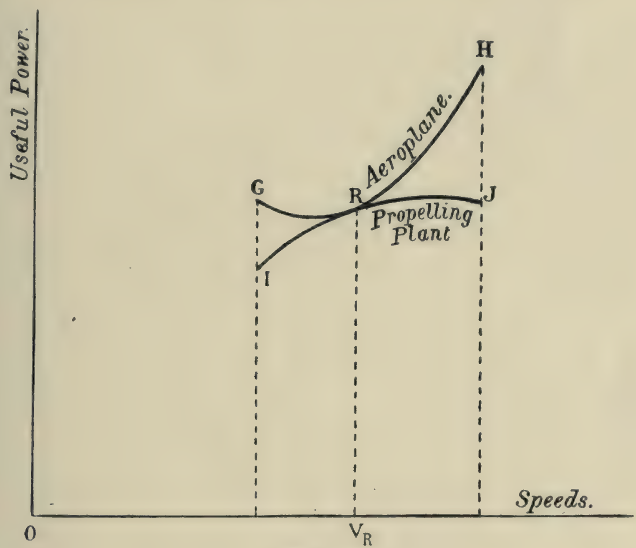

FIG. 21.

at a certain angle of incidence), to which correspond the two points M and M′ on the two curves.

The value of the maximum excess of power is determined by the distance MM′.

The maximum excess of useful power leads to several important results in connection with the application of the propelling plant to the aeroplane.

Let GH be the characteristic curve of a given aeroplane (Fig. 23), and on this curve let L be the point corresponding to the critical speed.

In order that the aeroplane might be sustained at this critical speed, for the least expenditure of motive power, the propelling-plant curve IJ would have to be of the shape shown in Fig. 23 (with the motor working at full power), that is, the highest point of the curve IJ, corresponding to the maximum efficiency, would have to be precisely the point L. The maximum excess of useful

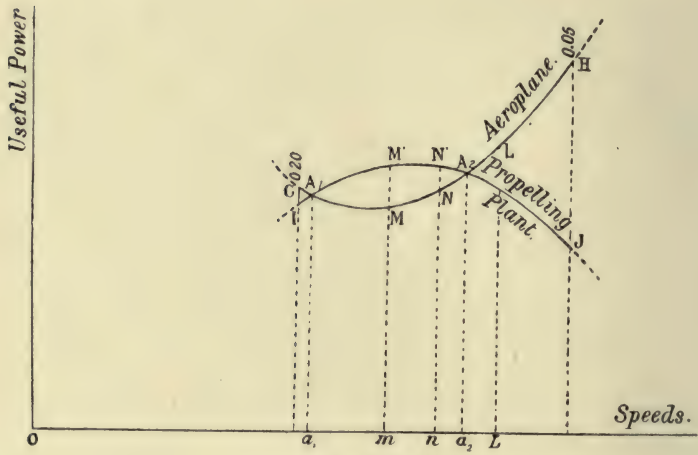

FIG. 22.

power available would then have the value MM′; and it would be obtained at the speed Om.

If it were desired to increase the excess of useful power available, the motive power would have to be increased. This could be accomplished in two ways:

(a) By providing the aeroplane with a propelling plant whose curve (with the motor running at full power) is of the shape of the curve I_1J_1 in Fig. 24. In this case the maximum excess of available useful power would be the greatest possible when the highest point of the propeller curve was situated *above* the lowest point of the aeroplane

curve. This particular arrangement[1] is shown in the curve I_1J_1 (Fig. 24). The maximum excess $M_1M'_1$ would then be obtained at a speed, O*m* (the economical speed), *entirely by the operation of the elevator.* But normally the propelling plant would not be running at its maximum-

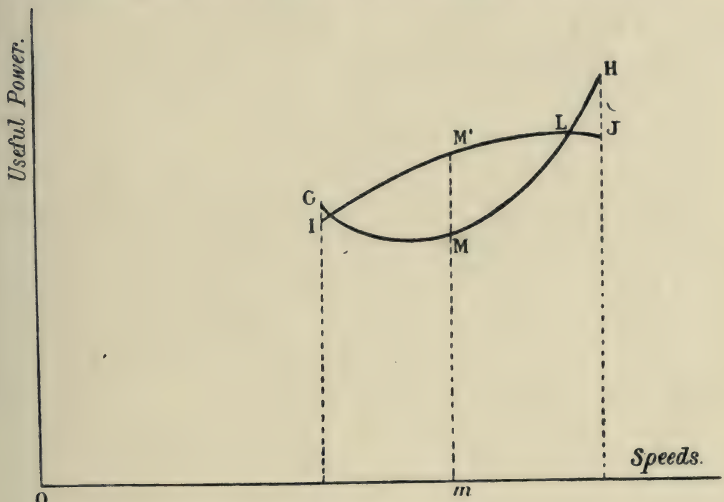

FIG. 23.

efficiency speed, since the point L representing the normal speed is not the highest point in the curve I_1J_1.

(*b*) Provide the aeroplane with a propelling plant whose *full power* curve has the form I_2J_2 (Fig. 24), but run the propeller at part power in order not to exceed the critical speed. In this case the point L would be *below* the curve

[1] The general case has not been included in this fig., in order to avoid complication. In the general case the highest point of the curve IJ would not be situated above the lowest point of the curve GH. The curve IJ would always pass through the point L. It may be readily seen by drawing the curve IJ (of the same height as the curve I_1J_1), that the maximum excess of useful power it renders attainable must always be inferior to that obtained from the arrangement I_1J_1. This will be shown more clearly in the discussion of the curve I_2J_2, which constitutes another special case of the general one.

I_2J_2. The maximum excess of useful power would then have the value $M_2M'_2$, and would be obtained at a certain speed, Om_2, *by means of a double manœuvre: the operation of the elevator and opening out the throttle.*

If the propelling plant in each case is supposed to be capable of developing the same maximum useful power (if, for instance, it consists in each case of a motor of the same H.P. driving a propeller of similar efficiency), the

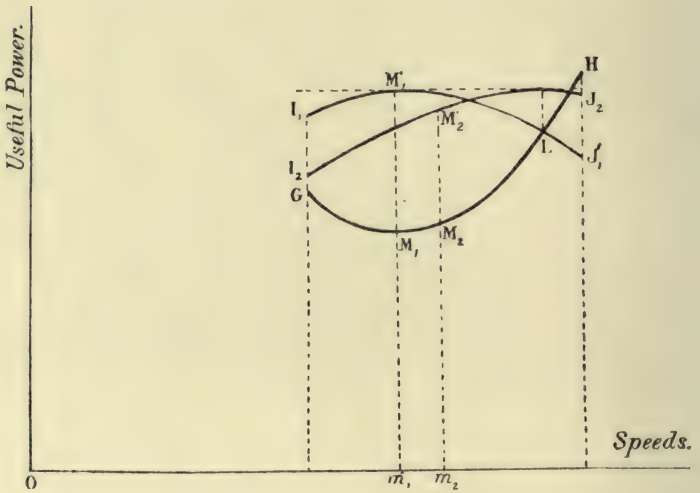

FIG. 24.

curves I_1J_1 and I_2J_2, which have been purposely included in the same figure, are equal in height.

Comparison will show that the curve I_1J_1 which corresponds to the motor working at full power, provides a greater reserve of power $M_1M'_1$ than is provided ($M_2M'_2$) by the curve I_2J_2 corresponding to the motor throttled down. This result may be interpreted as follows: in order to obtain the same excess of useful power in certain circumstances *it is necessary to employ, throttled down, a motor*

of greater power than would be the case if it were run all out.

Again, it is more advantageous to run the motor normally at full power, thus partly sacrificing the efficiency of the propelling plant, the while reserving its full efficiency for the moment when it will be necessary to make use of its excess of power, rather than to run the motor normally throttled down, and to make use when necessary of the excess power produced by running it at full power.

As stated in § 25, constructors as a general rule equip their machines with engines capable of developing greater power than is required merely for horizontal flight. Although the procedure answers one purpose, it would seem desirable not to exaggerate this tendency, for it is obviously unreasonable to provide for a reserve of power, that can at most only be required momentarily, by burdening the aeroplane with excess power (which means excess weight), which is not called upon throughout normal flight.[1]

There is room for the belief—as shown by the foregoing consideration—that, by judiciously adapting the propelling plant to the aeroplane, this necessary increase in the power of the motor may be reduced to a minimum.

By adopting the method illustrated by the curve I_2J_2, the propelling efficiency is partly sacrificed and the motor run at a speed slightly greater than its normal speed, which corresponds to its maximum power (§ 28).

Hereafter, however (see Part IV.), in making numerical calculations to set out the relations that must exist between the values of the aeroplane's characteristics, and the constructional data of the propelling plant, it will be assumed, so as not to complicate the calculations, that the propeller

[1] This stricture would lose its force if it became possible—and there is no reason why it should not in future—to diminish the area *in flight* (see §§ 6, 9, and 22), thus transforming the reserve power available into speed, without altering the angle of incidence.

works normally at full efficiency, and that both motor and propeller have been adapted to the conditions indicated in § 29.

31. Influence of a variation of the aeroplane's characteristics on the values of its speed, and of the thrust in horizontal flight at constant power.

Lastly, we have to consider the effect exerted on the speed and thrust values by a modification of one of the characteristics of an aeroplane running all out, that is at approximately constant power (§ 28). This point has been alluded to at the end of §§ 6, 9, and 15.

The first case to be examined will be that of an alteration in the weight, which often arises in practice when, for instance, an additional passenger is carried.

In Fig. 25, let GH be the curve of an aeroplane of weight P, area S, lifting efficiency K, and fineness f.

When the weight is given a value P′ greater than P, the other characteristics remaining constant, the aeroplane curve becomes G′H′.

This curve lies wholly above GH. In fact, on referring to formula (15a) of § 26:

$$T_u = \frac{1}{75}\left(\frac{P^2}{KSV} + \frac{KSV^3}{f^2}\right),$$

it will be seen that the values of the useful power which correspond to the same value V of the speed are greater, the greater the weight P.

The difference $T'_u - T_u$ between the values assumed by the useful power when the weight increases from P to P′ is expressed by $\frac{P'^2 - P^2}{75KSV}$, and grows smaller as the speed V increases. The curves GH and G′H′, as shown in Fig. 25, lie closer together in proportion as the curves are farther away from the vertical axis passing through O.

If the propelling plant curve IJ is brought on to the

same Fig., its points of intersection R and R′ with the curves GH and G′H′, fix the values V_R and $V_{R'}$ of the normal speeds[1] corresponding respectively to the values P and P′ of the weight. From Fig. 25 it is seen that the speed $V_{R'}$ is smaller than the speed V_R. Thus:

By increasing the load carried by a given aeroplane driven by a given propelling plant running at full power,

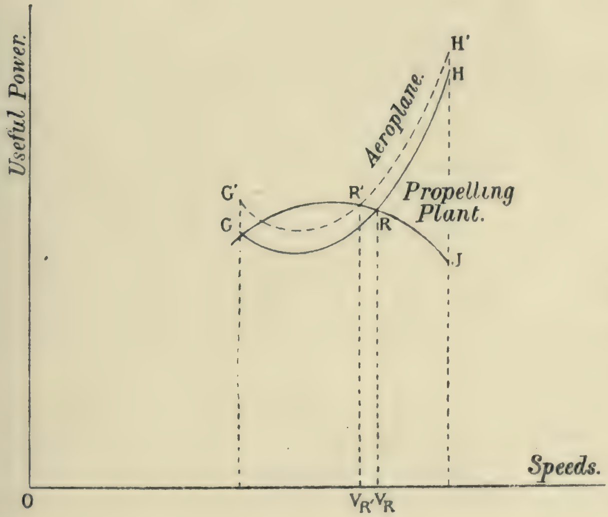

Fig. 25.

the speed is reduced, but more slightly according as it was originally greater.

As stated in § 9, this conclusion differs from that reached by considering the effect of an increase in the weight on the value of the speed in horizontal flight, *always supposing the angle of incidence to remain constant.* For in this latter case, in order to maintain the

[1] These speeds are assumed to be less than, or at most equal to, the critical speed of the aeroplane.

angle constant, it is necessary to increase the power, and thereby the speed.

Returning to the problem previously under consideration, it is further seen from Fig. 25 that an alteration of the weight has but little effect on the value of the useful power, more especially so if the point R is close to the highest point of the curve IJ. In the general case, besides, this value would tend rather to increase than to decrease, since as a rule the point R is situated to the right rather than to the left of the highest point of the curve IJ.

Since the value of the speed diminishes, that of the thrust must increase, owing to the fact that their product —being proportional to the value of the useful power— varies little, but always in the direction of an increase.

In a similar manner the effect of an alteration of the plane area, the lifting efficiency, and the fineness might be examined in succession. But until the future has succeeded in producing aeroplanes with variable surface or lifting efficiency, the practical value of such a discussion is only secondary.

Besides, the problem is distinctly complicated, since any variation of the plane area brings about (see § 14) a consequent variation of the detrimental surface; but if the latter is assumed to be constant the question is considerably simplified.

The difference $T'_u - T_u$ of the values assumed by the useful power when the plane area passes from the value S to the value S′—being deduced from formula (15) instead of from formula (15*a*)—may be expressed as

$$\frac{P^2}{75KV}\left(\frac{1}{S'}-\frac{1}{S}\right).$$

If the area S′ is greater than the area S the difference is negative; further, the absolute value of this difference is less in proportion as the speed V is greater.

The curve G′H′ (Fig. 26), corresponding to the value

S′ of the area therefore lies wholly below the curve GH corresponding to the value S, and these curves fall closer together as they are situated farther away from the vertical axis passing through O.

The normal speed $V_{R'}$ corresponding to the point of intersection R′ of the curve G′H′ and the curve IJ of the propelling plant, is consequently greater than the normal

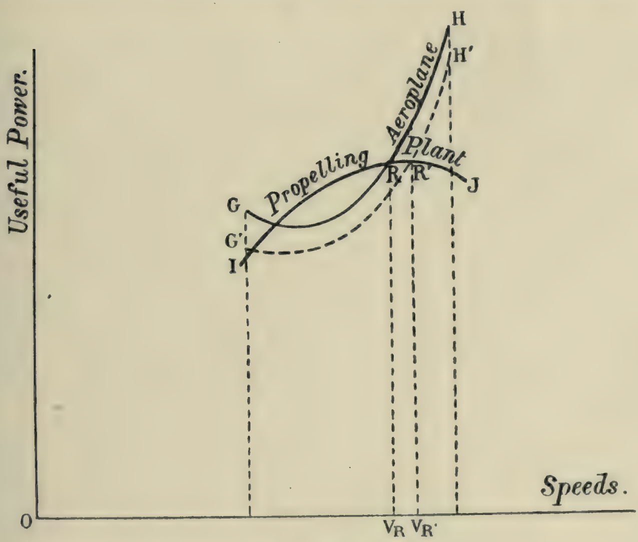

FIG. 26.

speed V_R corresponding to the point of intersection R of the curves GH and IJ. Thus:

By increasing the area of a given aeroplane driven by a given propelling plant working at full power, the speed is increased, but more slightly according as it was originally greater.

If the variation of the detrimental surface be taken into account, it will be found that the curve G′H′ lies even closer to the curve GH when the speed increases; it may even meet it and pass above it. For speeds exceeding a

certain value the results just obtained would consequently have to be reversed.

Regarding a variation of the thrust, finally, the same observation holds good. Since the value of the useful power remains approximately constant, the thrust varies inversely as the speed.

VI.—Summary of Chapter II.

32. Preliminary remarks.

In examining successively, in this chapter, the various factors affecting horizontal flight, chief stress has been laid on the predominant part played, in the flight of a given aeroplane, by the angle of incidence on which depend the values of the speed, the thrust, and the power required for sustentation.

Moreover, the effect on these values, exerted by the constituent parts of the aeroplane, termed its *characteristics*, has been specially studied from two points of view: (1) in comparing different aeroplanes; (2) in examining the flight—at constant power—of the same machine.

Finally, tables have been drawn up by means of which it is possible to calculate approximately these various effects, and to solve numerically every problem relating to horizontal flight.

At the end of this long chapter it has been deemed desirable to sum up the various questions studied, and, further, to lay down a few practical rules, easily remembered, for rapidly calculating the approximate values of the constituent elements—speed, thrust, and power—of the normal flight of an average aeroplane whose weight and area are known.

33. Summary of the chief properties of horizontal flight.

A.—In a given aeroplane the speed, thrust, and power entirely depend on the angle of incidence.

When the angle increases:

1. The speed diminishes.
2. The thrust at first diminishes, reaches a minimum at the optimum angle, and again increases.
3. The power at first diminishes, reaches a minimum at the economical angle, and again increases.

In a given aeroplane the angle of incidence is invariable, except when the motive power varies at the same time.

B.—When the other characteristics and the angle of incidence are assumed to be constant (in different aeroplanes or in the same machine with variable power)

(a) An increase of the weight:

1. Increases the speed proportionately to the square root of the ratio of increase (if the weight is quadrupled the speed is doubled).
2. Increases the thrust proportionately to the ratio of increase (if the weight is doubled the thrust is doubled).
3. Increases the power proportionately to the product of the ratio of increase and the square root of this ratio (if the weight is quadrupled the power is multiplied by 8).

And inversely.

(b) A reduction in the area of an aeroplane:

1. Increases the speed and the power proportionately to the square root of the ratio of reduction (if the area is reduced to one-quarter, the speed and power are doubled).
2. Does not affect the thrust.

And inversely.

(c) A variation in the lifting efficiency affects the speed, the thrust, and the power in the same sense as a variation of the area.

(d) An increase in the fineness of an aeroplane:

1. Does not affect the speed.

2. Diminishes the power and the thrust.

And inversely.

C.—When the other characteristics and the motive power are assumed to be constant (the angle of incidence being variable), an increase in the weight:

1. Diminishes the speed.

2. Increases the thrust.

And inversely.

34. Practical rules for calculating the elements of horizontal flight of an average, present-day type, aeroplane.

It may be simply stated, in the light of all that has gone before, that the values of the speed, of the thrust, and of the motive power belonging to the horizontal flight of an average, present-day, aeroplane (whose fineness, therefore, is $\frac{1}{0{\cdot}08}$, lifting efficiency 0·4, propeller efficiency 50 per cent.) are, for optimum flight (at the mean optimum angle 0·08):

$$V_1 = 5{\cdot}6\sqrt{\frac{P}{S}}, \quad t_1 = 0{\cdot}16\ P, \quad T_1 = 0{\cdot}0239\ P\sqrt{\frac{P}{S}}.$$

These values in economical flight (at the mean economical angle $0{\cdot}08 \times 1{\cdot}732$, or about 0·14) are:

$$V_e = 4{\cdot}25\sqrt{\frac{P}{S}}, \quad t_e = 0{\cdot}184\ P, \quad T_e = 0{\cdot}0209\ P\sqrt{\frac{P}{S}}.$$

If we assume—as practice entitles us to do—that the normal speed V_R lies between the speeds V_1 and V_e and that the propeller efficiency is slightly more than 50 per cent., the following formulæ will give, for the purpose of rapid and approximately correct calculation, the values of the speed, the thrust, and the motive power:

$$(16) \qquad V_R = 5\sqrt{\frac{P}{S}} \text{ (in metres per second).}$$

(17) $$t_R = \frac{P}{6} \text{ (in kilogrammes).}$$

(18) $$T_R = \frac{1}{50} P\sqrt{\frac{P}{S}} \text{ (in horse-power).}$$

From this we may deduce the following practical rules:

I.—The normal speed of an average aeroplane, in metres per second, is equal to 5 times the square root of the loading in kilogrammes per square metre.

II.—The normal thrust of an average aeroplane is equal to one-sixth of its weight.

III.—The motive power required to sustain an average aeroplane, in horse-power, is equal to one-fifth of the product of the weight (in kilogrammes) multiplied by the square root of the loading (in kilogrammes per square metre).

By substituting in formula (18) for $\sqrt{\frac{P}{S}}$ its value $\frac{V_R}{5}$, extracted from formula (16), we obtain:

(19) $$T_R = \frac{1}{250} P V_R,$$

which enables us to calculate the motive power required to propel at a speed V_R an average aeroplane of weight P, that is, to solve the cardinal problem of vehicles of transport:

What power must be expended to transport a given weight at a given speed?

The solution of this problem is contained, in a form easily remembered, in the fourth practical rule:

IV.—The motive power required to drive an average aeroplane at a certain speed is equal, in h.p., to $\frac{1}{250}$th of the product of this speed (in

metres per second) multiplied by the weight of the aeroplane (in kilogrammes).[1]

This rule may be still further simplified as follows:

IV*a*.—One h.p. can transport 250 kilogrammes at a speed of 1 m. p. sec.

Examples of the application of these rules

Calculate approximately the elements of horizontal flight of an average aeroplane weighing 320 kg. and measuring 20 sq. m. in area.

The loading is 16 sq. m.; its square root is 4.

The normal speed of the aeroplane is therefore

$$5 \times 4 = 20 \text{ m. p. sec., or } 70 \text{ km. p. h.}$$

The thrust is $\frac{320}{6}$, or 53 kg.

The motive power is $\frac{320 \times 4}{50}$, or about 25·6 H.P.

Example 2.—**What should be the plane area of an average aeroplane weighing 500 kg. and driven by a 30 h.p. motor?**

The square root x of the loading must be such that

$$30 = \frac{500 \times x}{50},$$

that is, $x = 3$.

[1] The practical formula (19) enables us to calculate approximately the effect exerted on the value of the speed of an average aeroplane by the ratio of the weight of its motor to the weight of the whole machine, and, further, the effect on the speed value of the weight per H.P. of the motor. If m represents the ratio of the weight of the motor to the weight of the aeroplane (a ratio usually equal to about $\frac{1}{6}$), and p the weight of the motor per H.P. (about 2 kg. per H.P. at the present time), we can obtain, by introducing these values into formula (19),

$$V = 250\frac{m}{p}.$$

Giving m and p their respective values $\frac{1}{6}$ and 2, the value of V is found to be 20 m. p. sec. or 75 km. p. h., which is as a matter of fact the average value of the speeds attained by aeroplanes at the present day.

The loading would therefore be 9 kg., which would give a plane area of $\frac{500}{9}$ = about 55 sq. m.

Example 3.—**What should be the power of a motor to drive an average aeroplane weighing 500 kg. at a speed of 20 m. p. sec.?**

By applying the fourth rule, the power is

$$\frac{500 \times 20}{250}, \text{ or } 40 \text{ H.P.}$$

Example 4.—**What will be the speed of an average aeroplane weighing 500 kg. and driven by a 36 h.p. motor?**

If x represent the speed required, x must be such that:

$$36 = \frac{500 \times x}{250},$$

which gives for x a value of 18 m. p. sec., or about 65 km. p. h.

The table on p. 224 facilitates the speedy solution of such problems in cases where square roots have to be extracted.

CHAPTER III

OBLIQUE FLIGHT OF THE AEROPLANE IN STILL AIR

I.—General Consideration of Oblique Flight

35. Definition of oblique flight—Slope of the flight-path—Angle of incidence.

When the direction of forward motion of an aeroplane is not horizontal, but ascending or descending, its flight is said to be *oblique.* The angle of this direction to the horizontal is termed the *slope* of the flight-path. It is expressed by a decimal fraction, is positive in ascending flight and negative in descending flight. It will be represented by the symbol a.

As stated previously (§ 7), the aeroplane is so built that if, when flying horizontally at a certain angle of incidence, its flight-path becomes inclined for some reason or other, the whole machine swings through an angle equal to that of the slope, so that the relative air-current, which has now a fresh direction, still strikes the planes at the same angle as before; provided always that the aviator has not manipulated his elevator.

This is what is meant by the expression, "an aeroplane *lies on its flight-path*" (the cause of this property will be explained in § 52).

If the flight-path becomes nearly vertically downwards, the aeroplane is said to *dive.* If, on the contrary, it approaches a vertical direction upwards, the aeroplane is said to *rear.*[1]

[1] The French equivalents *piquer* and *cabrer* are often, though for no apparent reason, used in English publications. In addition to being fully as explicit, the terms *dive* and *rear* have the merit of English nationality.—*Translators.*

Thus the angle of incidence, in oblique flight as in horizontal flight, remains the angle formed by the equivalent flat plane with the air-current which meets it, that is, with the flight-path.

36. Speed in oblique flight.

In the following we will throughout assume that the angle of the slope in oblique flight is a small angle. As a matter of fact, in practice it nearly always is a small angle, or at any rate every effort is always made to keep it small, since an aeroplane is not constructed to fly at a very steep angle.

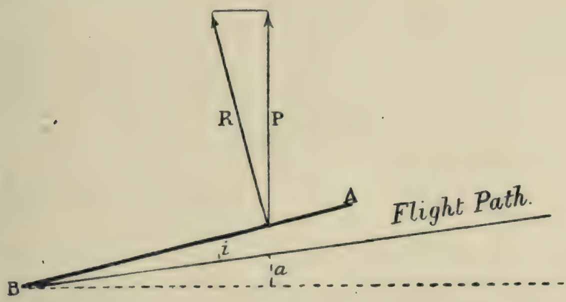

FIG. 27.

The reaction R of the air on the aeroplane (Fig. 27) may therefore, as in the case of horizontal flight, be considered near enough to a vertical direction[1] to allow us to assume it to be equal in magnitude to its component in this direction; and this component balances the weight of the machine.

Thus, the fundamental formula (4) may again be applied in the present case:

$$P = KSV^2 i,$$

[1] In descending flight, the reaction R is more nearly vertical than in horizontal flight; in ascending flight, on the other hand, it is farther removed from the vertical.

whence the following important conclusion, to which reference was already made in § 7, may be deduced:

Provided the angle of the flight-path is feeble, the speed assumed by an aeroplane in oblique flight is the same as that whereat it would travel in horizontal flight at the same angle of incidence.

A slight slope in the flight-path therefore has no effect on the value of the speed.

37. Power in oblique flight.

In horizontal flight the useful power required for sustentation (§ 18) is measured in kilogrammetre-seconds by the product Vt of the speed and the thrust; its value is $\mathrm{P}\left(i+\frac{1}{f^2 i}\right)$, where P, f, and i stand for the weight of the aeroplane, its fineness, and the angle of incidence.

In other words, the useful power required for horizontal flight may be written:

$$\mathrm{PV}\left(i+\frac{1}{f^2 i}\right),$$

If the aeroplane, travelling a distance V in one second, has risen in the same time along a slope a, it has climbed a height Va.

The work thus expended, in kilogrammetres, is PVa, and since it was expended in one second, the expression PVa also measures, in kilogrammetre-seconds, the additional power required for the ascent.

The total useful power required for oblique flight up a slope a, therefore finally amounts to:

$$\text{(20)} \qquad \mathrm{T}_u=\mathrm{PV}\left(i+\frac{1}{f^2 i}\right)+\mathrm{PV}a.$$

Since the whole of this useful power must be furnished by the propelling plant, and since the latter's efficiency is assumed as previously to be 50 per cent., oblique flight

along a slope a requires the expenditure of motive power equal in H.P. to:

$$(21) \qquad T_m = \frac{1}{37{\cdot}5} \text{ PV } \left(i + \frac{1}{f^2 i} + a\right).$$

This simple formula, assuming that the angles i and a are small, is applicable to ascending and descending flight. In the latter case, the slope a, of course, has a negative value.

II.—Gliding Flight

38. Gliding flight defined.

Examination of formula (21) shows that in descending flight, when the slope a gradually assumes more and more pronounced negative values, the power required for flight at a given angle of incidence i gradually grows smaller until it becomes zero at the slope:

$$(22) \qquad a = -\left(i + \frac{1}{f^2 i}\right).$$

Thus when left to itself, with its motor stopped, the aeroplane takes a descending flight-path inclined at a certain angle of slope a. This is known as *gliding flight.*

39. Variation of the slope of gliding flight with the angle of incidence in a given aeroplane—Maximum slope—Maximum range.

From formula (22) it will be seen that, in the case of a given aeroplane, or at any rate if f is constant:

The value of the slope of gliding flight depends only on that of the angle of incidence.

It will be remembered that the latter value, in its turn, depends only on the relative position of the various organs of the aeroplane, and the elevator in particular.

Here again, formula (22), like its parent formula (21), can only be applied if the angles i and a are small—a

condition which is satisfied if the angle of incidence lies between the limits 0·05 and 0·20.[1]

The absolute value $i+\frac{1}{f^2 i}$ of the slope reaches a minimum when the angle of incidence has a value $\frac{1}{f}$ (which is the value of the optimum angle i_1), since it is the sum of two factors whose product is constant and equal to $\frac{1}{f^2}$. Thus:

The optimum angle is the value of the angle of incidence that enables an aeroplane to glide at the gentlest slope, that is, to travel the greatest horizontal distance for a given loss of altitude.

The value of the minimum slope is therefore:

$$a_1 = 2i_1,$$

or double the value of the optimum angle.

The maximum range corresponding to a given loss of altitude z is:

$$\frac{z}{2i_1} \text{ or } \frac{zf}{2}.$$

The same results may be reached in a different manner. For, if equation (22) is written in the form:

$$i^2 - ai + i_1^2 = 0,$$

it becomes an equation whose roots have the values:

$$i = \frac{a \pm \sqrt{a^2 - 4i_1^2}}{2}, \tag{23}$$

so that:

$$a^2 - 4i_1^2 > 0, \text{ or } a > 2i_1.$$

The value $a = 2i_1$ is therefore the minimum slope.

It is clear, furthermore, that to each value a of the

[1] For an aeroplane of average fineness $\frac{1}{0{\cdot}08}$, the gliding slopes corresponding to the angles of incidence 0·05 and 0·20 would be respectively 0·18 and 0·23.

slope correspond two values i' and i'' of the angle of incidence, deduced from formula (23), and such that:

$$i'+i''=a, \quad \text{or } i'i''=i_1^2 .$$

Hence, a single path may be followed in gliding flight at two different angles of incidence whose sum is equal to the slope of the path, and whose product is equal to the square of the optimum angle of the aeroplane.

40. Speed in gliding flight.

Since the fundamental formula:

$$\text{(4)} \qquad P=KSV^2i$$

is applicable to oblique flight at a small angle, and therefore to gliding flight, the same thing is true of formula (5) directly derived from it:

$$\text{(5)} \qquad V=\sqrt{\frac{P}{KSi}}.$$

Now, it has just been seen that to each value a of the slope of the flight-path correspond two angles of incidence, i' and i'', the one greater, the other smaller than the optimum angle i_1. Substituting these two values in formula (5), we obtain two values for the speed, V' and V'', the one greater, the other smaller than the optimum speed V_1. Thus:

The same flight-path may be followed, in gliding flight, at two different angles of incidence and at two different speeds. When the slope of the flight-path is a minimum the two speeds and the two angles coincide.

These characteristic features of gliding flight may be represented by a curve such as shown in Fig. 28. This curve simply indicates the positions occupied, after the lapse of one second, by a machine starting to glide from O at the different usual angles of incidence.[1]

[1] One assumes that the aeroplane takes up its proper path from the point O (see § 41).

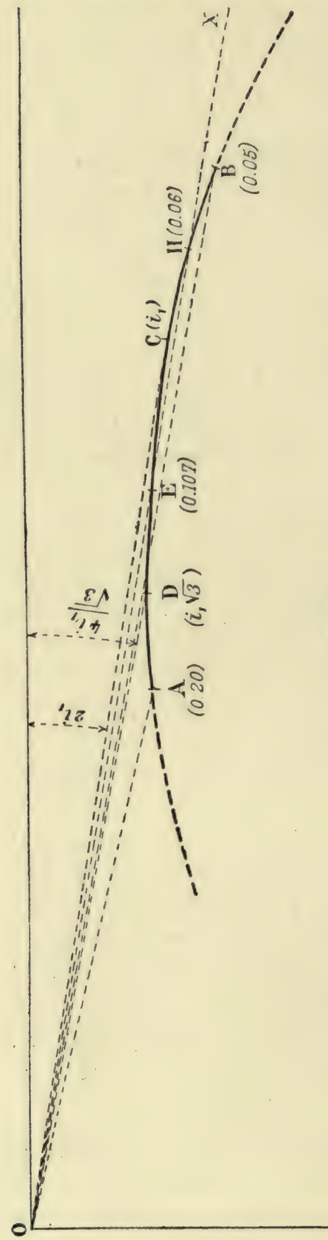

FIG. 28.

In other words, OA and OB represent in length and in direction the speeds assumed by the aeroplane at these angles of incidence.

It will be seen that to one flight-path — OX for instance—correspond two values, OE and OH, of the speed. At various points on the speed curve is indicated the corresponding value of the angle of incidence. The part of the curve plotted in a thick line, from A to B, corresponds to gliding flight at the usual angles of incidence from 0·05 to 0·20.

The point C corresponds to the minimum slope $2i_1$, and to the optimum angle i_1. The straight line OC is tangential to the curve at C, which proves that the two values of the angle of incidence and those of the speed coincide for this flight-path.

At the point D the tangent to the curve is horizontal. Therefore, by following the path OD, the glider loses least altitude in one second; in other words, its vertical or falling speed

will be least. In this case, therefore, the work produced, in unit time, by the weight of the aeroplane, or, to use another mode of expression, the mechanical power produced by the force of gravity is minimum. It will be easily understood that this result is obtained by utilising the economical angle of incidence i_e, which is equal to $i_1\sqrt{3}$.

The same result is derived from calculation, since the minimum rate of fall in one second (expressed by Va) will take place if V and a are given their values, as a function of i, derived respectively from formulæ (5) and (22), corresponding to the value i_e of the angle of incidence. It is further clear that, in this case, the slope a_e has the value $\frac{4i_1}{\sqrt{3}}$.

By taking these results as the basis of further calculation (which, however, is too long to be set out here), it is possible to complete the curve in Fig. 28 so as to include the case where the slope of the flight-path is no longer at a small angle (Fig. 29).

The dotted portion AM corresponds to gliding flights made at large angles of incidence from 12° to 90°. The lowest speed OM[1] is that of the parachute fall, which is, however, impossible in practice (see footnote 1, p. 146).

The dotted portion BLN corresponds to the very small angles of incidence, that is, to the *dangerous* angles. The greatest speed ON is that of the vertical dive.

The fact that the portion BLN of the curve is by far the most extensive clearly shows the important effect of the very slightest variation in the angle of incidence when the latter is small. In such a case even a minute diminu-

[1] This speed is, indeed, the lowest, but would not—if practicable—give the slowest rate of fall. As seen previously, this is given by the flight-path OD corresponding to the economical angle of incidence. In any case, the differences in the rate of fall are only slight for the usual angles of incidence.

tion of the angle causes the flight-path to approach the vertical. And, as the aeroplane always "lies down" on its

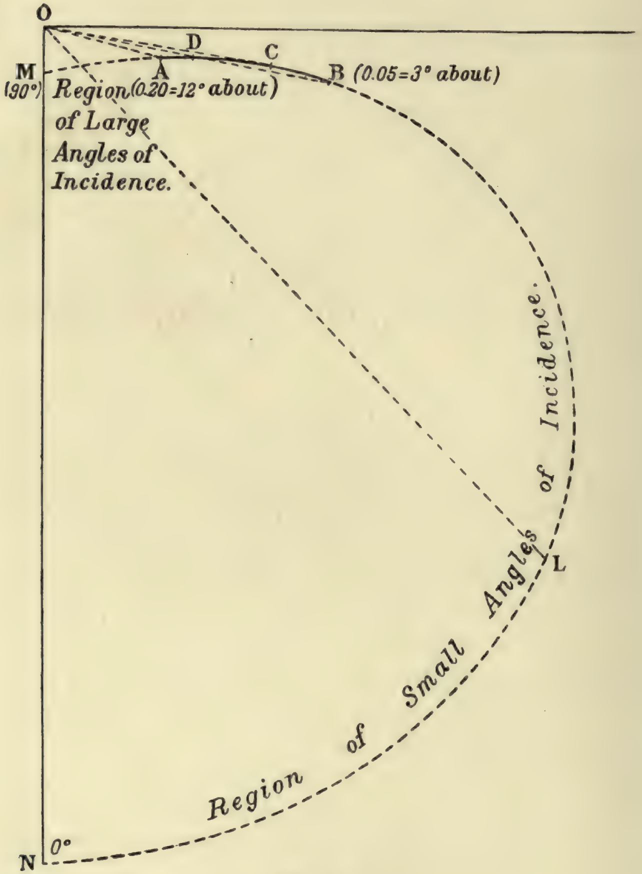

FIG. 29.

flight-path, a very slight decrease in the angle of incidence suffices to make the machine dive.

This shows the danger of seeking to obtain high speed in propelled flight by adopting a very small angle of incidence: if the motor should stop, the machine may

suddenly dive unless the aviator immediately works his elevator to increase the angle of his planes.

Having regard to this, it may be desirable to place the propeller axis slightly above the centre of gravity, this arrangement having the effect (see § 53), in case the propeller stops, of altering the longitudinal equilibrium of the machine relatively to its flight-path by automatically increasing the angle of incidence, without requiring the pilot to work his elevator.

41. Effect of the value of the characteristics of the aeroplane on that of its gliding slope.

In formula:

$$a = -\left(i + \frac{1}{f^2 i}\right) \qquad (22)$$

the only characteristic of the aeroplane included is the fineness. This leads to the following conclusion:

The gliding slope followed by an aeroplane of given fineness depends neither on its weight nor on its plane area.

At first sight this conclusion may well appear astonishing, more especially in so far as it concerns the weight. It must be distinctly understood, in the first place, that what is referred to is only the slope followed by the gliding aeroplane *once this gliding path has been regularly established.* For, at the start of its descent, a glider when it is abandoned to the air from a position of rest begins to follow a curved path (Fig. 30), until the moment when it attains and keeps to the slope of its final rectilinear gliding path.

If several aeroplanes *of the same fineness* but of the most divergent dimensions—whether small paper gliders or full-size machines—are thus liberated, the heaviest ones will, it is true, descend more steeply than the lighter ones before they enter upon their final straight gliding-path; but, provided these various machines move at the same

angle of incidence, this final gliding-path will have precisely the same slope in every single case; and herein resides the importance of our previous statement.

The only respect in which the flight of these various machines can differ is in their speed, and this is dependent on their loading.

With regard to the effect of the plane area, we have once again to emphasise the distinction that had to be

FIG. 30.

made in the discussion of the thrust in horizontal flight (§ 15):

It is only *by assuming the fineness to be constant* that we are entitled to assert that the slope of the gliding-path is independent of the plane area. The discussion in question might, in fact, well be reproduced integrally in the present case on account of the analogy of formulæ (22) and (11). The fineness, after all, is the only characteristic of the aeroplane whose value, *for an equal angle of incidence*, affects the slope of the gliding-path. The greater the fineness of the aeroplane, the smaller the slope of its gliding-path.

Aeroplanes possessing the greatest fineness are the best gliders.

It may be recalled that the fineness of an aeroplane is greater according as the detrimental surface is smaller, the

plane area more extended, and the lifting efficiency better. Hence:

Aeroplanes with large wing area and good lifting efficiency are, other things being equal, the best gliders.

The value of the minimum gliding slope $a_1 = 2i_1$, or the value $\frac{zf}{2}$ of the maximum range corresponding to a loss of altitude z, enables us to determine the gliding quality of a given aeroplane.

For a Wright biplane the value of the minimum gliding slope would be 0·12, or about $\frac{1}{8}$; the maximum range therefore would be about eight times the loss of altitude.

For a cellular Voisin biplane the value of the minimum gliding slope would be about 0·20, or $\frac{1}{5}$; the maximum range would be about five times the loss of altitude.

For an average present-day type of aeroplane, that is, for an aeroplane of the fineness $\frac{1}{0{\cdot}08}$, the value of the minimum slope would be 0·16, or about $\frac{1}{6}$; and the maximum range equal to about six times the loss of altitude.

After the foregoing, it appears almost superfluous to point out the error of believing, as some do, that an aeroplane may be able to sustain itself without motive power in still air. However perfect a glider may be, it must yet remain inexorably subjected to the force of gravity; that is, it must descend if no other force is opposed to gravity.

The error just alluded to can only have been entertained by reason of the fact that certain species of birds can sustain themselves in the atmosphere for long periods of time without the slightest wing beat. It will be seen hereafter (§ 75) that these birds probably extract from the movements of the air itself the power required to sustain them. As a matter of fact, there is no reason why the

aviators of the future should not be able to capture and utilise the same sources of natural energy.

42. Table for the rapid calculation of the gliding slope of an aeroplane.

Table III., which was drawn up (§ 17) to enable rapid calculation of the thrust required to sustain an aeroplane, and is here reprinted, also serves for the rapid calculation of the absolute value of the gliding slope followed, for an angle of incidence between 0·05 and 0·20, by an aeroplane or glider of given fineness.[1]

TABLE III. (reprinted)

Gliding Slopes.[2]

Angles of Incidence.	Values of the Optimum Angle designating the Fineness.				
	0·06	0·07	0·08	0·09	0·10
0·05	0·1220	0·1480	0·1780	0·2120	0·2500
0·06	**0·1200**	0·1416	0·1666	0·1950	0·2266
0·07	0·1214	**0·1400**	0·1614	0·1857	0·2128
0·08	0·1250	0·1412	**0·1600**	0·1812	0·2050
0·09	0·1300	0·1444	0·1611	**0·1800**	0·2011
0·10	0·1360	0·1490	0·1640	0·1810	**0·2000**
0·11	0·1420	0·1545	0·1681	0·1836	0·2009
0·12	0·1500	0·1608	0·1733	0·1875	0·2033
0·13	0·1577	0·1677	0·1792	0·1923	0·2069
0·14	0·1657	0·1750	0·1857	0·1978	0·2114
0·15	0·1740	0·1827	0·1927	0·2040	0·2167
0·16	0·1825	0·1906	0·2000	0·2106	0·2225
0·17	0·1911	0·1988	0·2076	0·2176	0·2288
0·18	0·2000	0·2072	0·2155	0·2250	0·2355
0·19	0·2089	0·2158	0·2237	0·2326	0·2426
0·20	0·2181	0·2245	0·2320	0·2405	0·2500

[1] Incidentally, this shows that:

The thrust required for the horizontal flight of an aeroplane is equal to the product of its weight and the slope it would follow in gliding flight at the same angle of incidence.

[2] The values of the minimum slope are given in heavy type. It will be seen that this minimum is, in fact, realised by using the optimum angle as the angle of incidence, and that its value is (as stated in § 39) equal to double that of the optimum angle.

In order to obtain the result sought for, all that is required is to take the number situated at the intersection of the line and column corresponding respectively to the values of the angle of incidence and of the fineness (optimum angle).

The warning made in § 17 regarding the exactness of the numbers in Table III. was of a general nature, and of course applies with equal force to the present case.

Example of how to use this table

Calculate the absolute value of the gliding slope followed, for an angle of incidence of 0·11, by an aeroplane of the fineness $\frac{1}{0{\cdot}09}$.

The number in the table, situated at the intersection of line 0·11 and column 0·09, is 0·1836. The value of the gliding slope is therefore approximately 0·184.

43. Part played by the motive power in horizontal flight.

A knowledge of the properties of gliding flight gives a correct perception of the part played by the motive power in the flight of an aeroplane. The following remarks have, in any case, already been partly set forth in § 7.

A given aeroplane left to itself when its motor stops will glide down along a path whose slope depends only on the value of the angle of incidence. The speed wherewith it travels along its flight-path remains the same as if it were flying horizontally or in oblique flight along a small slope at the same angle of incidence.

If, while gliding down, the pilot switches on his engine again, and gradually increases the power, the flight-path gradually resumes an ascending tendency, returns to the horizontal, and even, provided the power be sufficient,

exceeds it; *but the speed does not vary,* if the angle of incidence remains the same. Thus:

The function of the motive power is simply and solely to balance and overcome the action of gravity, and has no direct effect on the speed.

It may also be said that the part of the motive power is also to alter the direction of gravity, although it only does so in an imaginary way.

In fact, if an aeroplane glides down the slope OY (Fig. 31), and if the engine, being switched on, tends to

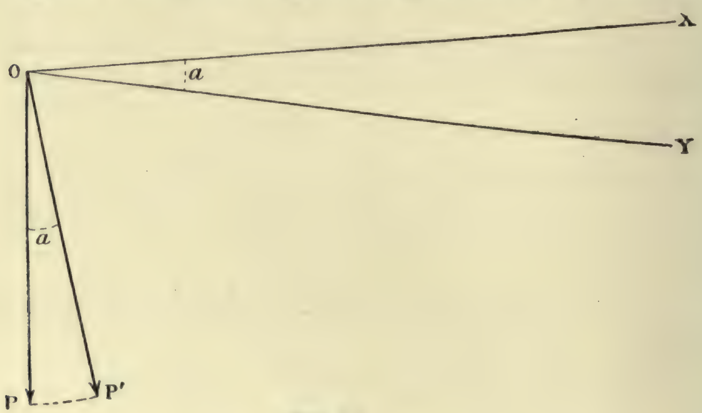

FIG. 31.

raise the flight-path along OX, for instance, the flight along OX may be considered as a glide through a surrounding medium in which the force of gravity acts in the direction OP′, making an angle a with the vertical OP.

From the foregoing it is absolutely clear that any excess power has the result of making the aeroplane ascend, while any lack of power causes it to descend, *but without altering the speed;* this is the direct opposite to what takes place with terrestrial vehicles which are forced to move on a fixed surface.

The flight of an aeroplane therefore constitutes a perpetual equilibrium of forces, and any irregularity in

the motive power tends to alter the direction of flight, which can only be maintained by a constant modification of the angle of incidence.

On the other hand, the importance of any irregularity in the working of the motor should not be exaggerated; it is only slight (as will be made clear in § 44), and the part played by the elevator in maintaining equilibrium would be unimportant if it were not for the constant irregularities of the wind (§ 75).

III.—Starting and Alighting

44. Ascending flight in practice—Climbing.

As it has already been stated several times that any excess power causes the aeroplane to ascend, without altering its speed, an increase in the motive power therefore affords a ready means of passing from horizontal to ascending flight.

This presupposes, of course, that the motor is not normally working at full power; so that the above method is not usual.

But there exists another method, which can be adopted even when the motor is working at full power, that is (§ 29), at constant power. The method is extremely simple and is the one usually adopted by aviators. It consists in operating the elevator.

From Fig. 32 (which is Fig. 22 of § 30 reprinted), containing the useful power and speed curves GH and IJ of the aeroplane and propelling plant respectively (the motor working at full power), it will be seen that—as the normal speed in horizontal flight of an aeroplane is V_R, which is measured by the length Or and the corresponding angle of incidence i_R—if this angle of incidence be increased to i_n, the useful power required to sustain the aeroplane diminishes and assumes the value Nn, instead of Rr. Since the useful power developed by the propelling

plant has, in its turn, become $N'n$, which is greater than Nn, there is an excess of power, and the aeroplane ascends.

It would, in fact, continue to ascend indefinitely, the angle of incidence still preserving its value, i_n, if the rarefaction of the air—which has not hitherto been considered in order not to complicate the discussion—did not reduce

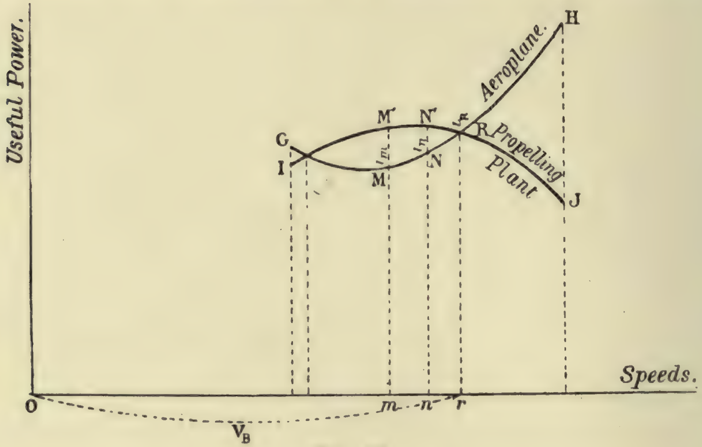

FIG. 32.

the value of the pressure on the planes and consequently their lift.[1] Thus:

By simply operating the elevator the aviator may alter the angle of his flight-path.

But this is only possible if, in normal horizontal flight, an excess of power is available, that is, so long as the curves GH and IJ intersect; if they are merely tangential (see § 30), the aeroplane would be able to fly horizontally at a single speed and angle of incidence, but it would be unable to ascend, and consequently would be unable to rise from the ground.

If the angle of incidence is still further increased, the excess of useful power continues to grow, and so too the

[1] The rarefaction of the air also affects the working of the propelling plant.

slope of the climb. This increase will continue until the angle of incidence has attained the value i_m, corresponding to the maximum excess MM′ of useful power. Therefore:

There exists an angle of incidence i_m the use of which gives the aeroplane its greatest climbing slope.

Beyond this definite value any increase in the angle of incidence diminishes the slope of ascent.

The considerations set out in § 30 have already made it clear that, to obtain the best results from a given motive power, the angle i_m should not be far removed from the economical angle i_e.

From formula (21), § 37:

$$(21) \qquad T_m = \frac{1}{37 \cdot 5} PV \left(i + \frac{1}{f^2 i} + a \right),$$

it is easy to calculate the slope along which an aeroplane can ascend for a given excess, h, of motive power (in H.P.).

This slope a must be such that $h = \frac{PVa}{37 \cdot 5}$, always assuming the propelling plant to have an efficiency of 50 per cent. Its value therefore is:

$$a = \frac{37 \cdot 5 h}{PV}.$$

Let us assume, in order to convey an idea of the importance of this limit slope in practice, that P and V have the average values 375 kg. and 20 m.p.s. Then:

$$a = \frac{h}{200}.$$

Hence, in order to cause an aeroplane weighing 375 kg. and travelling at 20 m.p.s. (that is, driven by 30 H.P. according to formula (19)—§ 34—to ascend along the feeble slope of 1 centimetre per metre (0·01), one requires an excess of power amounting to 2 H.P.

The above is at any rate reassuring in so far as it concerns the effect on the flight-path of any variations in the

motive power due to the irregular working of the motor. But it shows, on the other hand, that in order to ascend at a fairly steep angle, as is often required in practice, the maximum excess MM′ of motive power must be quite considerable.

This fact may lead to the partial sacrifice (see § 30) of the efficiency of the propelling plant with a view to increas-

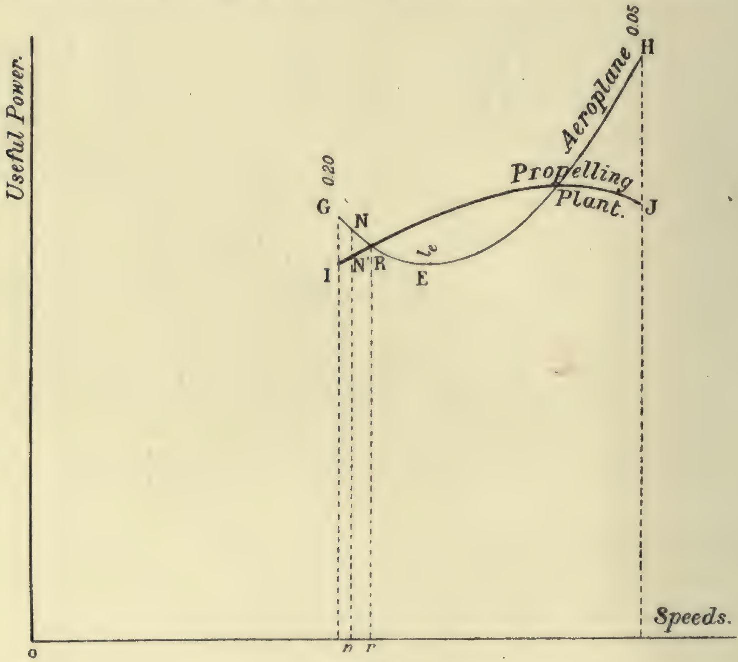

FIG. 33.

ing the value of the maximum excess of power available. In practice it also leads to the use of more powerful motors than required simply for horizontal flight, which is then effected with the motor throttled down.

The effect of the manipulation of the elevator may also be considered in the case where the point, on the curve GH, corresponding to horizontal flight of the aeroplane is

situated within the portion EG (Fig. 33), for instance at R, and when the aeroplane is, consequently, flying at an angle of incidence greater than the economical angle.

In this case any alteration of the angle of incidence brings about a lack of power; for, as the point on the curve is hereby brought back to N, the useful power Nn required to sustain the aeroplane is greater than the useful power N$'n$ actually developed by the propelling plant.

Here, therefore, it is true—however astonishing it may appear—*that an increase of the angle of incidence causes the aeroplane to fall instead of to rise.* Conversely, any diminution of the angle of incidence in such a case causes the machine to ascend.

This reversal of the usual effect of the elevator may be fraught with a certain amount of danger, more especially in starting (this will be further referred to in § 45). It is therefore desirable that *the angle of incidence should remain smaller than the economical angle.*[1]

45. Starting.

In order to rise from the ground, the aviator places his elevator in an attitude corresponding to flight at a very small angle of incidence, for the purpose of eliminating the active resistance or lift, and of reducing air resistance as far as possible to its passive part. The motor is then started, usually at full power.

As only the passive air resistance and the friction against the ground resist its forward motion, the aeroplane quickly attains a good rate of speed on the ground.

When the aviator deems the speed sufficient he inclines the elevator at an angle corresponding to flight at a fairly large angle of incidence. The machine then leaves the

[1] This consideration, already referred to in § 30 (footnote, p. 61), practically compels the choice, for the normal speed, of the greater of the two speeds possible in horizontal flight for a given aeroplane equipped with a given propelling plant working at constant power.

ground under the action of the lift, greater than its weight, created by its speed. Since the motor is working at full power, and as the friction on the ground has disappeared, the aeroplane is provided with a certain excess of power, and consequently continues to ascend.

If the machine rises too steeply, the elevator must be depressed; but this is a delicate manœuvre requiring experience, from lack of which arise the frequent mishaps in starting.

In order to rise after only a short run, the planes should be given a large angle of incidence.

The best angle, obviously, would be i_m, which gives the maximum excess of useful power available, and consequently the maximum slope of ascent (see § 44). But as it is difficult in practice to judge the precise attitude of the elevator corresponding to this value i_m of the angle of incidence, the aviator may easily make the angle of incidence too large.

Should the angle of incidence given to the planes be, in fact, greater than the economical angle, it may be necessary—as stated at the end of the preceding section—to reverse the usual operation of the elevator, otherwise the pilot, attempting to check the rapidity of ascent, is exposed to the danger of increasing still further the steepness of the slope.[1]

It need scarcely be pointed out that there is every advantage in starting head-on to the wind, since the speed of the wind is then added to the speed of the machine on the ground to produce the lift required for starting.

46. Alighting.

The aviator may alight either with his motor running

[1] This, however, is only possible if the motor is much more powerful than required to sustain the aeroplane. In such a case the best method perhaps is not to follow the maximum slope of ascent corresponding to the full motive power; in other words, always to sustain the angle of incidence below the value i_m, and keep the motor running at full power.

or in gliding flight. But in both cases his first object is to reduce his vertical speed, that is, his rate of fall.

With the motor working, the descent can be easily regulated by the combined use of the elevator and the throttle. In gliding flight, there is every advantage in descending at the economical angle of incidence which, as we have seen (§ 40), reduces the rate of fall to a minimum.

In either case, when near the ground, the aviator manipulates either the elevator or the motor so as to flatten out his flight-path and skim the earth. At the exact moment of landing it may be advisable to check the horizontal speed of the machine by increasing the angle of incidence, after the fashion adopted by birds, and pigeons in particular. The friction of the running wheels, or skids, on the ground plays an important part in checking the horizontal speed.

Finally, for reasons that will be explained in § 72, and in accordance with the dictates of common sense, the landing should always be made against the wind.

PART II

EQUILIBRIUM AND STABILITY OF THE AEROPLANE IN STILL AIR

CHAPTER IV

EQUILIBRIUM AND STABILITY IN STRAIGHT FLIGHT

I.—General Considerations

47. Preliminary Remarks.

As has been said at the beginning of this work, in order that the problem of aerial navigation by the aeroplane may be completely solved, it is not enough that the forward motion should generate a lift capable of supporting the machine; that is merely a beginning.

In flight, an aeroplane must be evenly balanced on its flight-path and, above all, the balance must be stable, that is to say, the machine must not upset or veer when subjected to a small disturbing influence.

That the question of the equilibrium and stability of the aeroplane may be considered in its true aspect, it is important, first of all, to examine carefully some postulates which follow:

1. An aeroplane moving through the air may, for the purpose of studying its equilibrium and stability, be considered as if, motionless and suspended from its centre of gravity, it were struck by a wind equal to that set up by its own speed.

This idea, in fact, is the interpretation of the mechanical

theory known as the theory of the movement of the centre of gravity.

2. As the oscillations of the aeroplane about various axes passing through its centre of gravity are slow on account of the considerable inertia of the machine, the reactions which the air opposes to these movements, if they are taken to be equal to those of still air, have a small and negligible value even if they occur with large surfaces,[1] although common error insists otherwise.

The first principle of the operation of all stabilising surface is the speed of the aeroplane itself.

It must not be forgotten that the machine is subjected in every part to the strong air-current, which it itself creates—a current which tends to keep in its proper course any surface opposed to it by exercising a force equal to the square of its speed. *This is the best basis for stability; the orthogonal resistance to oscillation goes for nothing.* At the same time, the rapidity of the oscillations, if appreciable, can be compounded with the speed of the aeroplane, so that the resultant, meeting the surfaces obliquely, deadens the oscillations and produces a braking effect, as will be shown later on (§§ 55 and 61).

3. There is another very widespread error that likens the equilibrium of the aeroplane to that of a floating body, a boat or balloon.

A floating body is, however, only subject to the vertical reaction of gravity. Its equilibrium can be represented diagrammatically (Fig. 34) by two vertical equal and opposite forces, P the weight applied to the centre of gravity G, and R, the reaction of the fluid, applied to a certain point C. When the body is gently moved from its position of equilibrium by rotation around its centre of gravity, the point C moves to C′, and R, *remaining*

[1] For example, a surface of 10 sq. m. striking the air at a speed of 1 m.p.s. only produces a stabilising force equal, by formula (1), § 1, to $0{\cdot}08 \times 10 \times 1 = 800$ grammes, which is clearly insignificant.

vertical, acts relatively to the point G, as a lever-arm G*g*, which, if the equilibrium is stable as in the case of Fig. 34, exercises a righting effect. But this cannot be applied to the aeroplane, because, when its equilibrium

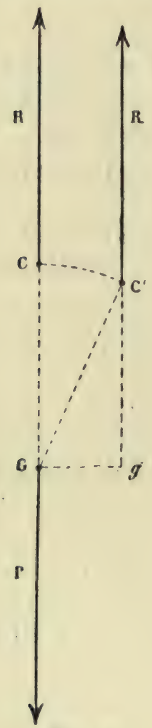

Fig. 34.

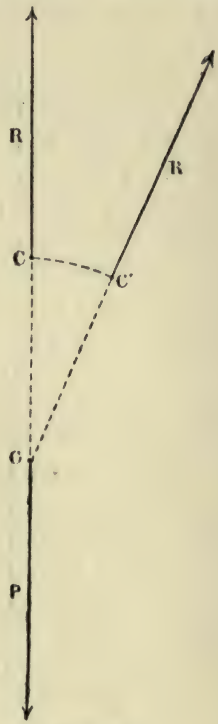

Fig. 35.

is disturbed, the reaction of the air R (Fig. 35), instead of keeping its original direction, *turns with the machine.* If, then, its point of application moves from C to C′, no righting lever arm is produced, as R still passes through the centre of gravity.[1]

[1] This is only true in the case of transverse oscillations in certain aeroplanes. In longitudinal oscillations, as we shall see (§ 54), the reaction R moves relatively to C′, thus forming what may be either a righting or upsetting lever arm.

48. The centre of pressure.

When an air-current strikes a vertical plane such as AB (Fig. 36), the pressure p, which is exerted on the plane, is applied to the symmetrical centre O. But when the plane is struck obliquely, the stream-lines follow paths similar to those shown in Fig. 37. A loss of speed (and perhaps even a backward motion) of the molecules of air, giving rise to a compression, results near the forward edge, while in the rear of the plane, where there is usually negative pressure, the stream-lines are only slightly affected.

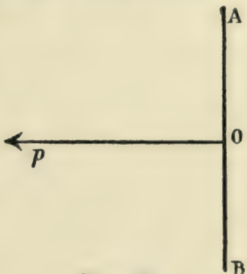

FIG. 36.

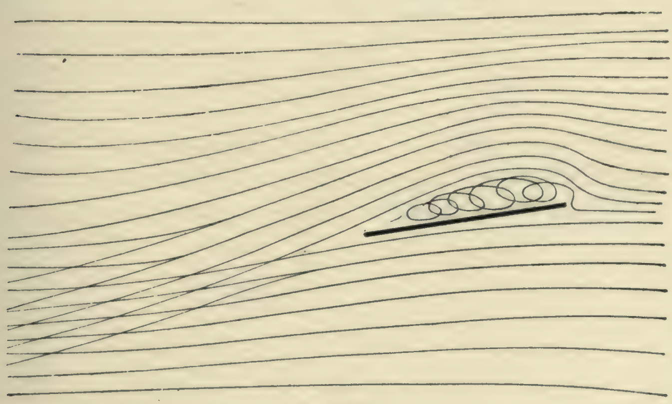

FIG. 37.

If from every point of a surface perpendiculars are drawn of a length proportional to the pressure at each point, a diagram is obtained similar to the lower portion of Fig. 38 (under surface).

Behind the plane, on the contrary, a negative pressure is produced wherein are eddies, and its distribution is represented on the upper part of Fig. 38 (upper surface).

M. Eiffel has shown by recent experiment that when

the angle of incidence of a flat plane is low, the value of the negative pressure on the upper surface of a plane is considerably more than that of the positive pressure

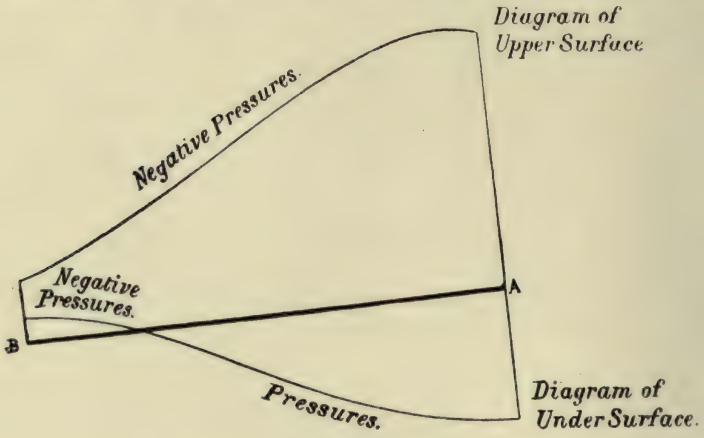

FIG. 38.

on the under surface. Thus, in this case it is the upper side of the plane that contributes most towards the creation of the lift, a function increasing as the angle grows

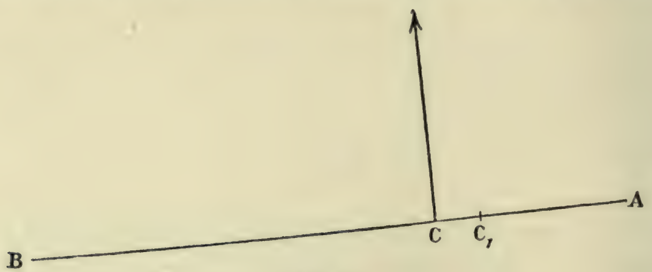

FIG. 39.

smaller. This fact shows that *the shape and smoothness of the upper surface of a plane have as much, if not more, importance, from the point of view of the value of pressure, as those of the under surface.* The result of the double action of the air-current with pressure in front and nega-

tive pressure behind, both unequally distributed, is that the total reaction on the plane is applied at a point C (Fig. 39), nearer to the forward edge A than to the trailing edge B. This point C is called *the centre of pressure* of the plane.

In a flat plane, C moves towards the forward edge as the angle of incidence becomes smaller until, when the

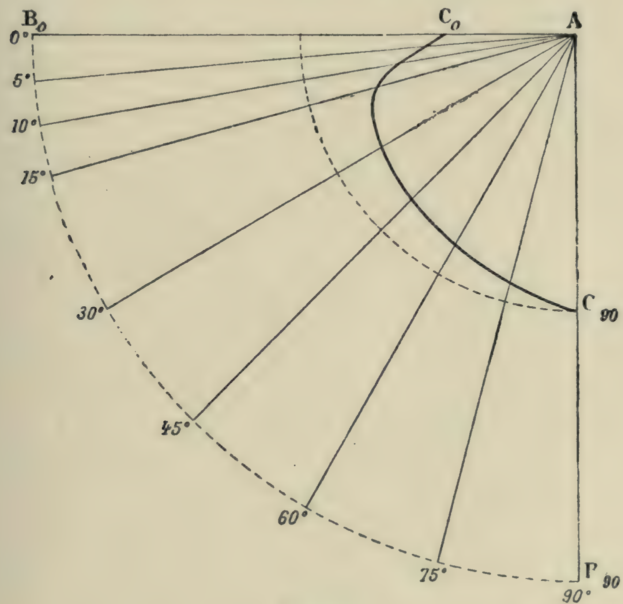

FIG. 40.

angle is zero, it reaches the limit point C_1—the distance between A and C_1 being equal, approximately, to $\frac{1}{4}$ of the fore and aft dimension AB of the plane.

M. Eiffel, from his recent experiments, has plotted a curve (see Fig. 40) showing the variations of the position of the centre of pressure on a flat plane as the angle of incidence moves from 0° to 90°.

For the usual small angles of incidence, and supposing

the variation to be small, it can be stated that the distance CC_1 (Fig. 39) is given by the formula

$$CC_1 = ali,$$

l representing the chord of the plane, and a a coefficient whose value, according to an old formula of Avanzini, is

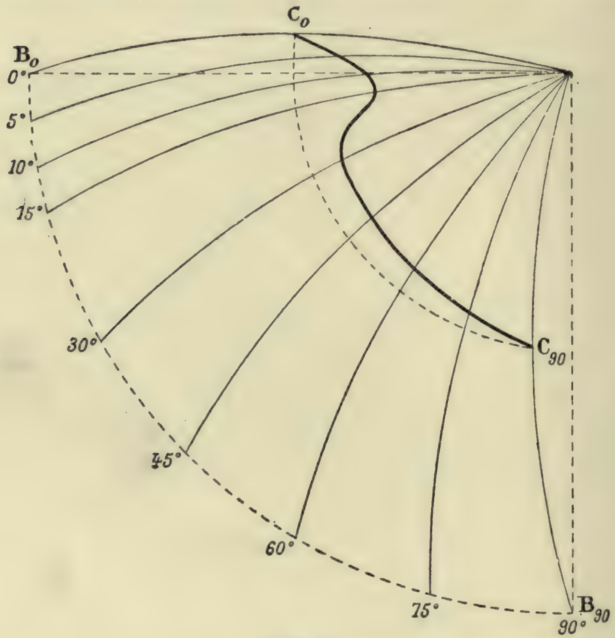

FIG. 41.

about 0·3. In the light of M. Eiffel's recent researches, the mean value of this coefficient is slightly greater—0·4 at least—for the usual angles of incidence. We will adopt this value, so that the rule governing the variations of the centre of gravity on a flat plane will be represented by the formula:

$$CC_1 = 0{\cdot}4li. \tag{24}$$

In curved planes the rule is, according to M. Eiffel, quite

different, and he has prepared a diagram (Fig. 41) to illustrate it (the angles of incidence being those of the chord). It may be seen that the centre of pressure, which is situated at the middle of the curve when the air-current is parallel to the chord, moves directly into the air-flow until it

FIG. 42.

reaches a definite point—regulated by the angle of the chord—and then moves back to its original position. Fig. 42 shows the pressure and negative pressure curves on the section of a curved plane, and Fig. 43 the stream-line flow.

As planes curved similarly to those used by M. Eiffel would in actual practice only be used at angles of in-

cidence smaller than that which corresponds to the forward limit point of the centre of pressure, it can be said that the variation of the centre of pressure in curves is inverse to that in flat planes.

But the curve, as previously mentioned (§ 3), is sufficiently deep to bring about this result. With the planes flatter it is possible that the forward limit point of the centre of pressure would be reached at a smaller angle of incidence, that is to say, at a usual value of the angle.

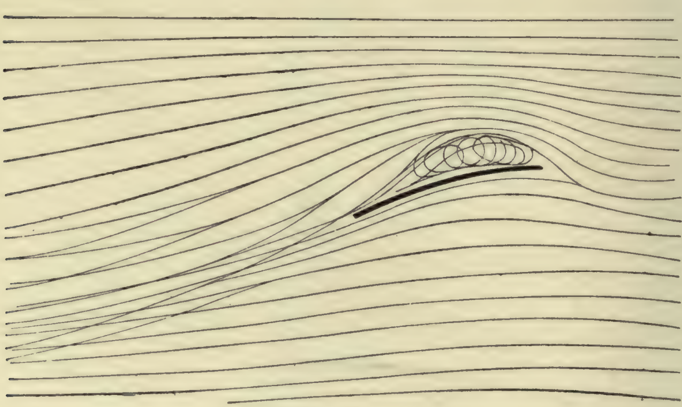

FIG. 43.

It is somewhat difficult, in this case, to determine precisely how the centre of pressure varies at different angles since, according as the angle at which this variation starts is either greater or less than the particular angle corresponding to the limit point, an equal variation in the angle of incidence will cause the centre of pressure to advance in the one case and retreat in the other. At any rate, the variations of the centre of pressure complicate the problem of aeroplane stability, and therefore it is important to reduce them. As they are proportional to the chord of the plane, it is advantageous to increase its span and reduce its chord, while keeping the same area. The same result can be

obtained by distributing the carrying surface among several planes, as the variations of the centre of pressure are more limited in a multiplane than in a monoplane of the same supporting area (presuming the planes to be of a similar shape).

49. The three kinds of stability.

In order to fly, the aeroplane must preserve on its flight-path equilibrium of three kinds:

1. Longitudinal stability,

that is to say, the least disturbing influence does not cause it to dive or rear.

2. Lateral stability,

that is to say, the least side inclination does not turn it completely over.

3. Directional stability,

that is to say that the least change in its direction does not cause it to veer, and that it moves always head-on to the wind it creates, without drifting to leeward.

Moreover, the rolling and pitching oscillations must not become so great that, as a result of an action similar to that of a pendulum, the aeroplane upsets.

II.—Longitudinal Stability

50. The longitudinal equilibrium of the aeroplane on its flight-path.

In all the following remarks, the tractive effort of the propelling plant, that is, the thrust exerted along the axis of the propeller, will be taken as passing through the centre of gravity of the aeroplane.

This condition has the effect of preserving equilibrium when the thrust is cut off by the aviator stopping the motor for a glide. (At the same time we may admit that the equilibrium is upset to a certain degree, and place the axis of the propeller a little above the centre of gravity.)

If an aeroplane consisting of one principal plane, or perhaps of several other planes besides, is supposed to remain rigidly in a certain position and a stream of air be directed perpendicularly to its main plane, the effect on all parts of the machine can be summed up in a single resultant situated in the plane of symmetry of the aeroplane.

FIG. 44.

Let R_{90} (Fig. 44) be the direction in which this resultant works. Then, the aeroplane remaining fixed, by altering the direction of the air-current so as successively to meet the main plane at decreasing angles i_1, i_2, &c., and finally at zero, a different resultant of the air pressure is obtained each time. Let R_{i_1}, R_{i_2}, . . . R_0 be the directions of these various resultants, which can be imagined to be identical with and rigidly attached to the machine.

Now, supposing the aeroplane to be free in the air, in order that it may be in equilibrium on the flight-path, the three forces which affect it, *i.e.* its weight, the thrust of the propeller, and the reaction of the air, must, collectively, exercise no turning moment about the centre of gravity. As the first two forces pass through this centre, the reaction must do likewise. If, therefore, the centre of gravity, G, be outside the lines of the resultants (Fig. 44), it is not possible to find an angle of incidence of which the corresponding resultant passes through G. The machine is therefore unstable.

On the other hand, when the centre G coincides with one of the resultants, R_{i_n} (Fig. 45), which corresponds to

a certain angle of incidence, i_n, the three said forces pass through the centre of gravity, consequently inducing stability at this point. In such a case the aeroplane will be stable only when the angle of incidence has the one value i_n. We may conclude as follows:

An aeroplane in which the planes are fixed relatively to one another can only fly at a single angle of incidence. Therefore it is essential for one, at least, of the planes to be movable.

The movable plane is usually called the *elevator*. It will be considered at length in § 52.

For every fresh position of the movable plane the lines of the resultants change, and it is no longer the resultant R_{i_n}, corresponding to the angle of incidence i_n, but the resultant $R_{i_{n'}}$, corresponding to the angle of incidence $i_{n'}$, which passes through the centre of gravity.

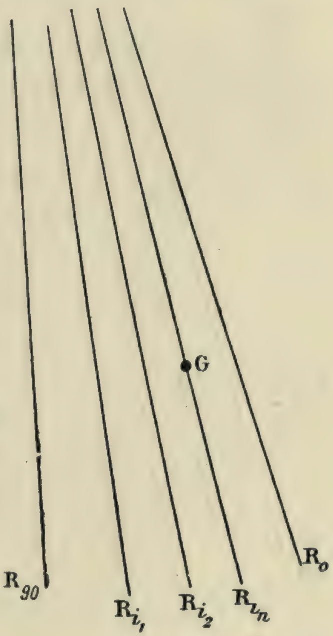

FIG. 45.

Therefore it is possible by altering the angle of the movable plane or elevator to cause the aeroplane to assume on its flight-path a series of different stable attitudes corresponding to the different values of the angle of incidence.

At the same time the centre of gravity must occupy a definite position, for if it is not possible by altering the angle of incidence to find a resultant which passes through the said centre of gravity, stability is unobtainable, and *the aeroplane will have been badly designed.*

The action of the elevator does not generally permit of all angles of incidence ranging from 0° to 90°, so that **stability can only be achieved by the values of the angle of incidence between two fixed limits.**

It is evident, too, that the angle of incidence corresponding to the normal speed of the machine is included in the scale of angles, without which the machine is badly designed.

The foregoing considerations show, also, that **the angle of incidence can be altered by altering the position of the centre of gravity.**

51. Example of longitudinal stability—The stability of a rectangular plane.

Let AB (Fig. 46) be a rectangular plane, the length, AB, being 100 m/m., according to the scale of the diagram. The forward limit point of the centre of pressure is at C_1, situated from the leading edge at a distance AC_1, equal to ¼ of the length of the plane, that is 25 m/m. The pressures will be taken as entirely normal to the plane AB.

If, then, a current of air is directed perpendicularly upon it, the resultant, that is the pressure R_{90}, passes through the centre O of the figure. As the angle between the current and the plane is diminished the pressure approaches its limit point R_0 (which passes through C_1) and reaches it when the angle of incidence is zero.

The resultants considered in the preceding section are the same, in this single case, as the perpendiculars to the line AB between R_{90} and R_0. One sees, moreover, that if the centre of gravity, assumed to be situated on the plane AB, is outside the region OC_1, it will not be possible to find for AB a stable position on its flight-path.

But if the centre of gravity G is placed between the points O and C, stability is possible, as a resultant R_{i_n}, corresponding to the angle of incidence i_n, passes through this

point. And in order that this angle of incidence i_n may be included between the practical limits 0·05 and 0·20, the resultant R_{i_n} must itself be included between the result-

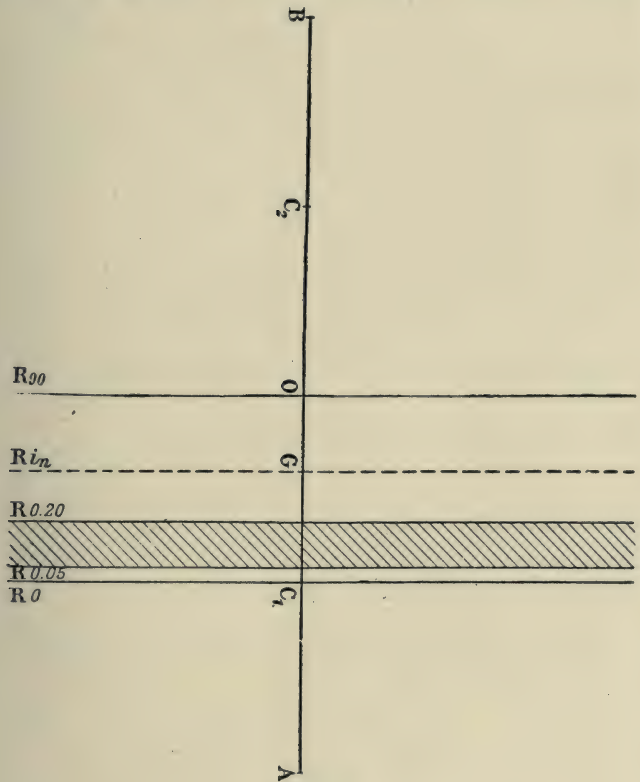

FIG. 46.

ants $R_{0·05}$ and $R_{0·20}$ situated respectively, according to formula

(24) $$CC_1 = 0·4li,$$

at the distance 0·02AB and 0·08AB from the resultant R_0, that is, in the present example, 2 m/m. and 8 m/m.

Thus to each position of the centre of gravity there

corresponds one angle of incidence, and only one; and in order that this angle may be confined within practical limits the centre of gravity must be placed in the shaded portion of Fig. 46, that is to say, between two points distant less than $\frac{1}{15}$ of the length AB from each other.[1]

52. The tail—The elevator.

The foregoing considerations show that for flying at different angles of incidence without altering its centre of gravity, an aeroplane must consist of at least two planes, one fixed and the other movable.

Let two such planes be taken, as in Fig. 47, which are are at a small angle x to one another and a short distance apart. Now when the air strikes the leading plane AB at the angle i it exerts upon it the pressure $R_1 = K_1S_1V^2i$ (S_1 = the surface, K_1 = the total resistance, lift, and drift of the plane).

The second plane is struck at the angle $i - x$, and taking the symbols S_2 and K_2 as representing respectively its surface and its total resistance, the pressure R_2, which it sustains, equals $K_2S_2V^2\ (i-x)$.

To achieve stability, the product of each of the pressures and its distance from the centre of gravity, in other words, the *moment* of each pressure in relation to this point, must be the same:[2]

$$R_1 \times Gg_1 = R_2 \times Gg_2,$$

or

$$K_1S_1V^2i \times Gg_1 = K_2S_2V^2(i-x) \times Gg_2.$$

[1] This conclusion conveys some idea of the sensitiveness of a device whereby the angle of incidence of a single flat plane might be varied by altering the position of the centre of gravity. With a plane 2 m. in length, it would be enough to move the centre of gravity 12 cm. in order to cause the angle of incidence to pass through the whole scale of its admissible values.

[2] This supposes (§ 50) that the thrust passes through the centre of gravity. If it did not, this force would produce a moment which would have to be taken into account in writing the equation of stability (§ 53). One must, moreover, assume that the head resistance of the parts of the aeroplane other than the planes also pass through the centre of gravity.

It is clear, firstly, that speed need not be considered in this equation, which shows that an aeroplane in equilibrium struck in a certain direction by a current of air travelling at a certain speed will preserve its equilibrium, even though the speed of the air-current changes, if the direction remains unaltered. Also in this equation the values of Gg_1 and Gg_2, which depend on the angle i, must be altered as the centre of pressure varies at different angles of incidence. (If the second plane be small and

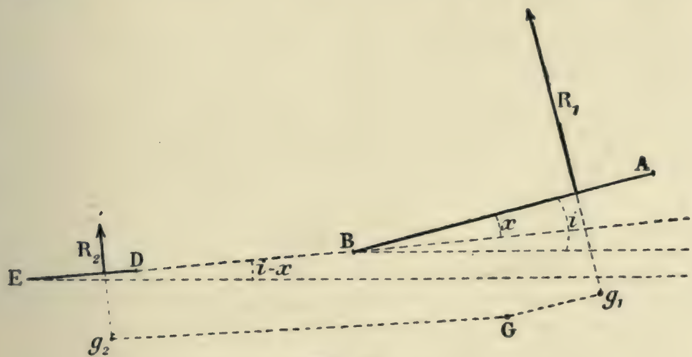

FIG. 47.

sufficiently far from the centre of gravity, one can neglect the variations of the centre of pressure and assume Gg_2 to be constant.)

One obtains, finally, a relation between the angles i and x, which is affected by certain characteristics and dimensions in the construction of the aeroplane, particularly those which fix the position of the centre of gravity with regard to the planes.

The existence of such relation clearly shows that the value of the angle x made by the two planes and that of the angle of incidence i—when the aeroplane is stable on its flight-path — correspond to each other. This result agrees with the deductions of a general character set out in § 50.

Moreover, it allows us to take into account the influence on the relationship of the values of the constituent parts of the aeroplane, especially those of the dimensions which fix the position of the centre of gravity. And, in particular, the reality of the factors of the equation in i—assuming x to be constant—shows the possibility of finding a solution of the problem.

All these calculations will not be shown here since they would take up too much space,[1] besides, it is chiefly the results that they give that are of interest. They show *the importance of having an auxiliary plane smaller than the main plane and placed at a certain distance from the centre of gravity in order to preserve the longitudinal stability.* Such plane is usually called the *tail.*

The tail, as a rule, comprises both fixed and movable parts.

The fixed tail is usually placed in rear and in such a way that the stable attitude of the machine resulting from its position and from its size corresponds to the single angle of incidence.

While very important in some machines (Voisin), the fixed tail is suppressed in others (Wright), but it should be noticed that such suppression cannot be absolute. The action of the air on the parts other than the planes always establishes—even though such parts were not so disposed with this object—a vertical component which is due to the thickness of the materials used, to the angle at which they are struck by the wind which the speed creates, and to the skin friction, &c. However little these components are removed from the centre of gravity, they interfere by their movement with the longitudinal stability in a way which, though slight, cannot be disregarded.

An aeroplane has therefore, of necessity, an *imaginary*

[1] A simple instance of this class of calculation is shown, however, by way of example, at the end of the present paragraph. (See footnote 2, p. 119.)

fixed tail, even if the constructor has not designedly allowed for it, in the same way as it possesses detrimental surface (§ 11).

If it is desired to represent this tail by a hypothetical surface, its size should vary with its supposed distance from the centre of gravity. The best way is to assume that the effect of the imaginary fixed tail is measured by a certain moment around this point.

The tail comprises besides its fixed part, real or imaginary, a movable part which is an essential, for, as has been seen, it enables the aeroplane to fly at various angles of incidence.

This movable part, termed the *elevator*, is placed sometimes in front and sometimes at the rear.[1] Ordinarily its movements are made about an axis perpendicular to the plane of symmetry.[2]

There is, for every position of the elevator, one stable position of the aeroplane, and one only, that is to say, one angle of incidence.

It can be mathematically determined what conditions must be fulfilled so that the operation of the elevator has a correct influence, neither too much nor too little, on the value of the angle of incidence. These conditions obviously depend on the size of the machine and on the relative positions (as well as on their form in section) of the planes, and also influence the value of the limits between which the angle of incidence may be varied by operating the elevator. Moreover, it is necessary that the practical values of the angle of incidence, that is, those comprised between the normal angle and the angle of incidence below which it would be dangerous to go, should

[1] The effect of a forward elevator is quicker, but also more abrupt, than that of the rear elevator.

[2] It seems possible that it will, later on, be found useful to use an elevator capable of sliding horizontally without rotating, or perhaps a fixed plane with a variable surface.

be included in the scale of angles, failing which the machine is badly designed.

The tail action makes it clear why it is said that the aeroplane *lies on its flight-path* (§§ 7 and 35).

As the elevator has a fixed position, the stability is only possible for the machine at one angle of incidence on its flight-path in whatever direction this path may lie. If, therefore, the aeroplane changes direction, it ought, as the angle of incidence is not altered, to tilt about the centre of gravity into its new flight-path.

The tail may or may not lift. As a rule, fixed tails do

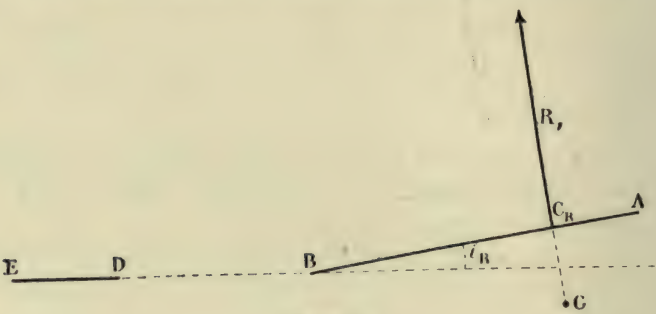

FIG. 48.

not; they are placed so as to present no angle to the wind when the main plane is at its normal angle of incidence (Fig. 48). In this case the centre of gravity should be projected from the point C_R, the centre of pressure corresponding to the normal angle of incidence, i_R, in order that the moments of the pressure on the main plane and the tail should be equalised, that is to say, eliminated. Later on it will be seen (§ 75) that this arrangement is advantageous in view of the wind effect.

In the case of the arrangement shown in Fig. 49, where the tail is struck on its upper surface, the projection of the centre of pressure lies in front of the point C_R, and will be discussed in §§ 54 and 75.

In the original Wright machine the single[1] tail, constituting the elevator, lifts. The stability, moreover, of

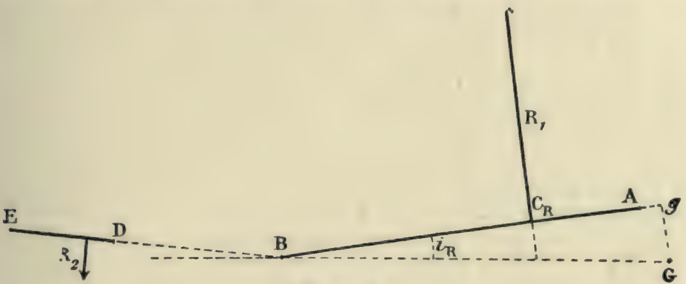

FIG. 49.

this aeroplane is peculiar, as the main plane is placed behind (Fig. 50). The projection, g, of the centre of

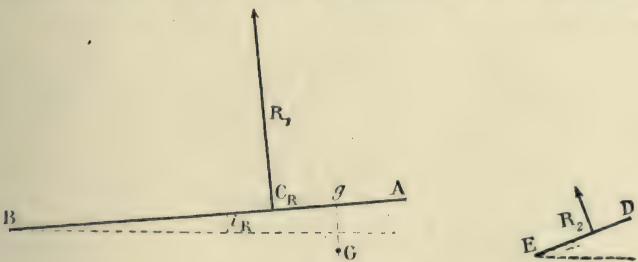

FIG. 50.

gravity must in this case lie in advance of the centre of pressure C_R.[2] Besides the function just defined, the tail plays the double part of an automatic stabiliser and of a damper-out of oscillations (§§ 54 and 55).

[1] Single, because the imaginary tail which is necessary in all aeroplanes is embodied in it, and at the same time the machine has no fixed tail.

[2] By way of example, it will perhaps be well to give an instance of the kind of calculation mentioned at the beginning of the present paragraph.

Let there be supposed an aeroplane consisting only of a main plane and a movable tail or elevator placed in rear (see Fig. 47). Further, to simplify the case, let the centre of pressure be taken as constant when the angle of incidence varies.

53. The direction and position of the thrust.

If the elevator occupies a fixed position, the aeroplane can only fly at one angle of incidence.

Then, if it is granted that

$$Gg_1 = g_1, \quad Gg_2 = g_2, \quad \frac{K_2S_2}{K_1S_1} = m,$$

the equation of stability will be

$$g_1 i = mg_2 (i - x), \quad \therefore i = \frac{mg_2 x}{mg_2 - g_1}.$$

It is clear, moreover, in order that the angle of incidence of the main plane may have a positive value that the inequality $g_1 < mg_2$ be satisfied. The position of the centre of gravity should not then be moved back beyond a certain limit point.

Furthermore, as was foreseen, the value of the angle of incidence i only depends on that of the angle x which the two planes make between them, that is to say, on the position of the elevator.

In the present example, the two angles in question vary proportionately one to the other, and their relation $\frac{i}{x}$ is measured by the value of the expression

$$\frac{mg_2}{mg_2 = g_1}.$$

When $g_1 = 0$, the angles i and x are equal. In this case any movement of the elevator affects only the value of the angle of incidence. The tail always remains parallel with the wind and does not lift.

When g_1 is positive, as in the case of a lifting tail, the angle of incidence varies more than the angle made by the two planes. It varies less when g_1 is negative, as happens in the case of the tail struck on its upper surface. If the planes are relatively placed in the position assumed

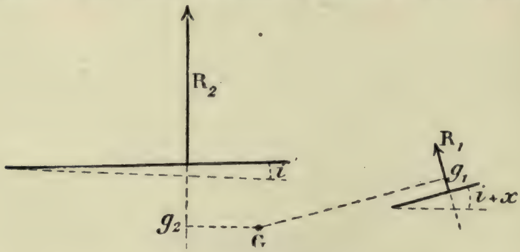

FIG. 50A.

in a Wright aeroplane (Fig. 50A), the equation of stability becomes—taking the symbol i to equal the angle of incidence of the main plane and assuming that

$$\frac{K_1S_1}{K_2S_2} = m,$$

$$mg_1 (i + x) = g_2 i, \quad \therefore i = \frac{mg_1 x}{g_2 - mg_1}.$$

This is true whatever the direction of flight, horizontal or oblique, and whatever the direction and value of the forces affecting the machine may be, particularly the thrust.

This last result may appear surprising, and it seems at first sight that the aeroplane ought to follow its propeller, that is, that the flight-path of the machine should naturally follow the line of the propeller's axis. It is not so, however, for if the axis (which for the moment we will suppose to pass through the centre of gravity) does not make with the main plane an angle equal to the single angle of incidence as fixed by the position of the elevator, the axis and the flight-path will never coincide whatever the latter may be; the two lines will always be at the same angle to each other (Fig. 51).

Any form of flight can be made—a horizontal flight for instance—by giving the axis of the propeller a certain inclination.

This happens when the aeroplane flies at an angle of incidence other than its normal angle; the plane and the axis of the propeller being immovable in their relationship, if the angle of the former increases with the direction of the flight-path, the angle of the latter to the said flight-path is increased to the same amount.

It is possible to consider the equilibrium of the aeroplane in horizontal flight in another way.

The necessary condition $g_2 > mg_1$ means, by analogy with the case taken above, that the centre of gravity should not be moved back beyond a certain limit point.

Again, according as the said centre of gravity were placed on either one side or the other of a second point, so that $g_2 = mg_1$, the angle of incidence will vary either more or less than the angle formed by the two planes.

Since these results are only approximate, as the shifting centre of pressure has not been taken into account, it should be plainly stated that the influence of the position of the elevator on the value of the angle of incidence may be regulated at will by the dimensions of the machine, and particularly by the situation of the centre of gravity in relation to the planes.

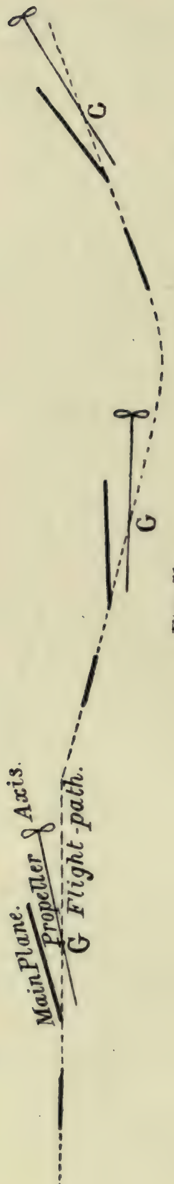

FIG. 51.

Let GX (Fig. 52) be the direction of that one of the resultants explained in § 50, which, corresponding to a certain angle of incidence i, passes through the centre of gravity. The aeroplane can only fly at this angle of incidence i. If it is supposed that the machine flies horizontally, the direction of the resultant GX will not alter. Also, let P be the weight of the aeroplane. If any straight line GY is drawn from G, it is always possible to resolve the force $-P$, which is equal and directly opposed to the weight of the machine, into two forces: t directed along GY, and R directed along GX.

Thus, for every direction of the thrust GY it is possible to find a value of it, t, and a value, R, of the reaction of the air on the aeroplane, which together sustain its weight.

Horizontal flight is then obtainable at the given angle of incidence i whatever the direction of the thrust may be; it is accomplished, for example, in each case at a certain speed V, so that the reaction R of the air has a value equal to KSV^2i.

In other words, if the thrust has a vertical component, that is, if it tends to lift a portion of the aeroplane's weight, the lift of the plane may be reduced by just that portion of the weight.

To reduce the lift the aeroplane should then go slower than when the plane supports the entire weight of the machine.

If the thrust has a vertical direction, the horizontal speed vanishes. The thrust should then be equal to the weight, and it will no longer be necessary for the plane, which thus becomes useless, to lift.

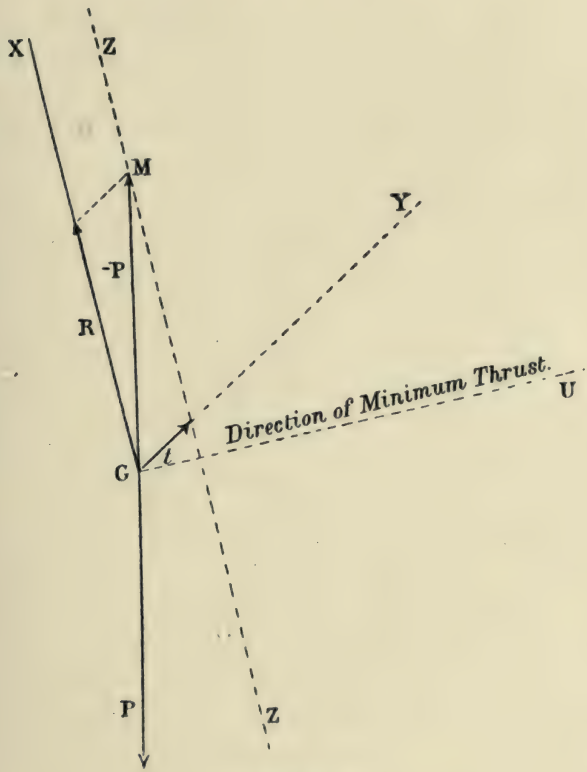

FIG. 52.

The machine would operate as a *helicopter*.

An examination of Fig. 52 shows that since the extremity of the component t is on ZZ, which is parallel to GX and met by the extremity M of the force $-$P, the direction in which this component, the thrust, offers the least value is along the perpendicular GU. And this

being at a slight angle from the vertical, one departs in practice from the conditions of the minimum thrust [1] by placing the propeller so that at the normal speed and angle of incidence of the machine its axis coincides with the flight-path.

It has been hitherto supposed that the axis of the propeller or, what is equivalent, the thrust, passes through the centre of gravity of the aeroplane.

If it does not (Fig. 53), the machine's position in longi-

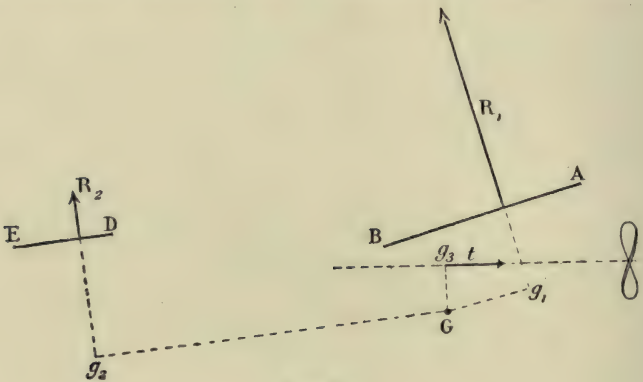

FIG. 53.

tudinal equilibrium will be altered, because the moment arising from this change is added—according as the thrust is above or below the centre of gravity—to one of the two moments set up by the action of the air on the tail and on the main plane. *Equilibrium could not then be obtained at the same angle of incidence.*

Such an arrangement destroys the equilibrium when the motor stops and the moment that the thrust ceases. But it can be advantageous, nevertheless, to adopt it, if it is desired that at the instant the spark is cut off the angle

[1] This minimum thrust must not be confounded with that which has been defined in § 12. It is the direction of the thrust which varies, the angle of incidence is assumed to be fixed.

of incidence should, without the elevator being moved, take on a value best fitted for the glide, allowing the slowest rate of fall, that is, the value of the economic angle (§ 40).

As the value of the normal angle of incidence is, however, the smallest, as a rule, that can be adopted without danger and corresponds to the speed-limit (§§ 25 and 27), the loss of equilibrium brought about by the motor stopping—if it should so happen—should have the effect of increasing this value rather than of diminishing it, and in consequence the axis of the propeller should be above the centre of gravity. This arrangement is used in the Wright machine; the distance between the axis and the centre of gravity is about 0·50 m.

54. Automatic longitudinal stability.

The aeroplane should have a position of longitudinal equilibrium on its flight-path; this equilibrium should be stable, so that when the machine is disturbed from this position, it should automatically return to it.

It has been seen in § 50, that an aeroplane, *supposedly stationary*, experiences, when struck by air-currents at different angles, reactions of which the resultant assumes a position varying with the angle at which the main plane is struck, and that, moreover, the resultant which passes through the centre of gravity corresponds to a single angle of incidence at which the aeroplane flies at a position of equilibrium.

Let R_{i_n} (Fig. 54) be this resultant passing through the centre of gravity G of the machine. *Continuing to assume this to be fixed,* when the angle at which the air strikes the machine increases to the extent of the very small angle i', the resultant of the reactions changes its position to $R_{i_n+i'}$, making with respect to the centre of gravity a lever arm Gg, which would tend, if the aeroplane were not fixed, to make it turn in a certain direction.

The same thing happens when the aeroplane, *instead of being immovable*, flies in equilibrium on its flight-path, and is subjected to a slight change of angle around its centre of gravity, which has the effect of increasing its angle of incidence. The *automatic longitudinal stability* or otherwise of the aeroplane depends entirely on the action of the lever-arm, according as it exercises a *righting* or an *upsetting moment*.

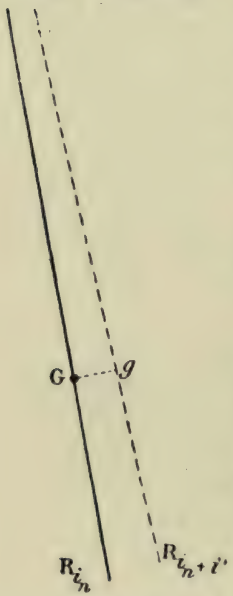

FIG. 54.

The consideration of the simple case already treated in § 51 will make this stability process more readily understood.

An aeroplane consisting of a single plane,[1] AB (Fig. 55), is in equilibrium on its flight-path when the pressure or resultant R_{i_n}, corresponding to a certain value i_n of the angle of incidence, passes through the centre of gravity.

When the angle of the plane is reduced by the small amount i', and moves from AB to A′B′, the pressure instead of continuing to pass through the centre of gravity moves forward (as we know occurs when the angle of incidence is diminished), and occupies the position $R_{i_n-i'}$, thus creating in respect to the centre of gravity a righting moment. (The result would be just the same if the angle of incidence were increased instead of diminished.) Therefore:

An aeroplane consisting of a single flat plane is automatically stable in the longitudinal sense.

[1] It will be supposed, in order to make the case as general as possible, that the centre of gravity lies outside the surface of the plane.

If this single plane were curved instead of flat, a lessening of the angle of incidence would have the effect, as has been seen, of causing the centre of pressure to move back. The moment of the resultant, $R_{i_n - i'}$, would then be an upsetting moment. So that:

An aeroplane consisting of a single curved plane is, usually, unstable in the longitudinal sense.

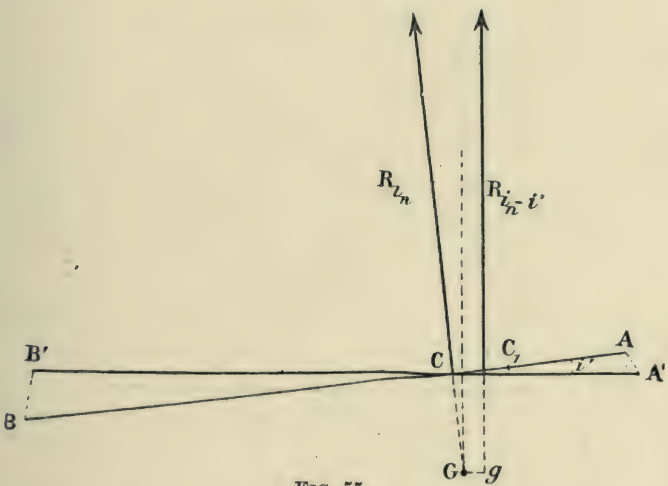

FIG. 55.

The problem of longitudinal stability is best understood by the aid of calculation. The method is simple. Taking first the general case of the aeroplane consisting of two planes, flat or curved, an examination afterwards of special cases will help the reader to grasp the results arrived at.

First of all the equation of stability is written, which in the general case under consideration is

$$K_1S_1V^2i_1g_1 = K_2S_2V^2i_2g_2.$$

The signs K, S, V, i have their usual meaning, the sign g represents the distance between the centre of pres-

sure and the centre of gravity. The index sign 1 applies to the forward plane, which may be the main plane, as it generally is, or the tail as in the original Wright machine; the index-sign 2 denotes the rear plane.

One proceeds to express the variation of each of the two moments producing equilibrium when both the angles of incidence, i_1 and i_2, vary by the same small angle i'. The difference between these two variations is the same as the value of the righting or upsetting moment set up by the angular change i'.

As a rule, the values of the two factors which compose the normal moment of the plane—the pressure and the leverage—are both influenced by an alteration of the angle of incidence. The value of the pressure is doubly influenced, since it depends, at the same time, on that of the angle of incidence and on that of the speed, which latter also varies with the angle.

But supposing that the aeroplane flies oscillating rapidly through a small arc, it may be granted, on account of the inertia of the mass, that the speed has not time to be affected appreciably, and therefore it remains constant.

If the centre of pressure did not move, the variation of the normal moment of the front plane might be expressed by $K_1 S_1 V^2 g_1 i'$.

But the centre of pressure does move, though like the angular change i' the movement is very small, and can be considered proportional to i' and measured by the product $a_1 l_1 i'$, the sign l being the fore-and-aft dimension of the plane and a a coefficient, positive or negative according as the plane is flat or curved, and according to its position relatively to the centre of gravity.

The variation of the moment[1] due to the travel of the

[1] The variation of the value of the pressure can here be disregarded as it would merely introduce a negligible term i'^2.

centre of pressure is therefore: $K_1S_1V^2i_1 \times a_1l_1i'$, and the total variation of the normal moment of the front plane is

$$K_1S_1V^2g_1i' + K_1S_1V^2i_1 \times a_1l_1i',$$

or

$$K_1S_1V^2i'(g_1 + a_1l_1i_1).$$

Similarly the variation of the normal moment of the rear plane is:

$$K_2S_2V^2i'(g_2 + a_2l_2i_2).$$

The difference between these two variations, the formula for which is too long to write here, represents the value of the moment set up by the resultant of the air-pressure when the aeroplane gets off its normal angle to the extent of i'.

In order that the longitudinal equilibrium may be automatically stable, this must be a righting moment, and pursuing the formula, it is clear that such a condition is realised when the inequality[1]

$$i_1 - i_2 > i_1i_2\left(\frac{a_1l_1}{g_1} + \frac{a_2l_2}{g_2}\right)$$

is satisfied.

The examination of special cases will show exactly the sense in which this condition of stability should be interpreted.

In the first place, if it be supposed that the centre of pressure does not travel as the angle of incidence varies, the coefficients a_1 and a_2 can be made algebraically of no value, so that the said condition of stability becomes:

$$i_1 - i_2 > 0, \quad \text{or} \quad i_1 > i_2.$$

In this case, for the aeroplane to be stable the angle of incidence of the front plane (whether it is the tail or main plane) must be larger than the angle of incidence of the

[1] In the latter part of this formula one of the two products $\frac{al}{g}$ which concerns the tail is generally negligible, so that the movement of the centre of pressure on the tail can be practically disregarded.

rear plane—in other words, *the planes must form in relation to one another a dihedral angle or* **V** (Fig. 56). This dihedral angle need not be very far removed from 180°, at which figure the equilibrium is indifferent.

When, in the inequality expressing the general condition of stability, the second portion, $\left(\frac{a_1 l_1}{g_1}+\frac{a_2 l_2}{g_2}\right)$, is positive, as is usual in the case of curved planes, the first portion must, *a fortiori*, be positive also.

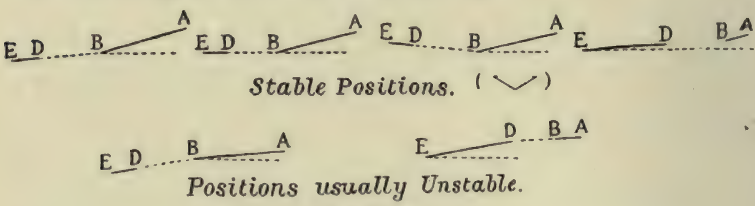

FIG. 56.

In order that an aeroplane with curved planes may be stable, not only should the planes form a dihedral angle, but this angle must not pass a certain limit[1] or it will be unstable.

On the other hand, when the second part of the inequality is negative, as is usual in the case of flat planes, the aeroplane is stable when the planes are in the same straight line, and also when they form a slight inverted dihedral angle. In this last case, which is essentially theoretical, stability will be most precarious.

We can generalise, then, by saying that:

The main plane and the tail of an aeroplane should form between them a dihedral angle or V, otherwise the machine is unstable.

The degree of longitudinal stability of an aeroplane can, obviously, be measured by the relation between the

[1] This limit is about 180° as a rule. It should be noted, however, that as the angle of incidence of curved planes is not reckoned from the chord, the angle is really a little greater than it appears.

righting moment set up by a small change in angle and this angle itself. It would take too long to discuss the value, taking the relationship in the general case just stated, so only two very simple hypotheses will be examined here, one where the aeroplane consists of a single plane, and the other where it has a non-lifting tail.

In the first instance, the righting moment resulting from the change in angle i' is expressed, as formerly stated, as $KSV^2i \times ali'$, or, by substituting the weight P for the product KSV^2i, as $aPli'$.

The relation of this moment to the change of angle i' causing it, which characterises the degree of longitudinal

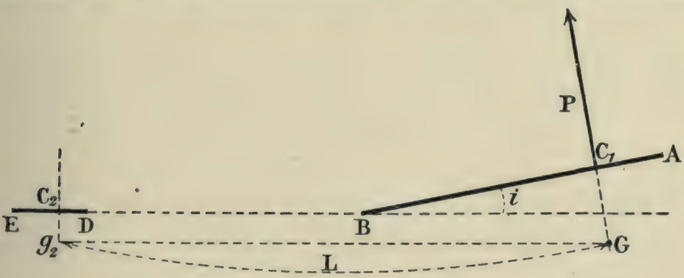

FIG. 57.

stability, bears the value aPl, where, in flat planes, $a = 0{\cdot}4$ (§ 48). So that an aeroplane composed of a single flat surface will be the more stable in the longitudinal sense as its weight and chord increase.

It would therefore be profitable to increase the chord of the planes and to diminish their span, which would reduce their lift. But it is clear that the use of a fixed tail renders unnecessary this disadvantageous method of increasing stability.

Taking an aeroplane with a non-lifting tail (Fig. 57), and considering the main plane separately, the change i' in the angle of incidence creates the principal moment with a value, as in the preceding case, of $aPli'$, a being positive if the plane is flat, and negative if it is curved.

Furthermore, the angle of incidence of the main plane causes the tail to deviate from the path of the wind, thus giving rise to a second moment, which is always a righting moment, of a value $KsV^2i' \times L$, s and L being respectively the surface of the tail and its lever-arm on the distance Gg_2 from the centre of gravity to a vertical line drawn through the centre of pressure C_2 of the tail (the variation of which can be disregarded). The coefficient K is, in the interests of simplicity, supposed to be the same for the tail as for the main plane.

If $\frac{P}{Si}$, taken from the fundamental formula[1] (4) is substituted for the product KV^2 and the relation $\frac{s}{S}$ between the surface of the tail and the main plane is represented by m, the formula of this second moment is:

$$\frac{PmL}{i} i'.$$

The righting moment due to the tail is, as has been seen, inversely proportional to the angle of incidence. Therefore:

The aeroplane is more stable longitudinally, the smaller the angle of incidence, that is, the higher the speed.

Moreover, the righting moment is proportional to the lever-arm L of the tail, and to the relation m, between the surface of the tail and that of the main plane.

Indeed, the stabilising action of the tail is of so much greater importance than that of the main plane that the latter, if not negligible, is entirely secondary. Figures give some idea of the relative value of these forces.

If, for instance, a flat-surfaced aeroplane is flying at an angle of incidence 0·1 and its main plane chord is 2 m., and the tail with a lever-arm of 4 m. has a surface equal to

[1] The tail surface is not included in the value S of this formula, as it is supposedly non-lifting.

$\frac{1}{10}$th of the main plane, the righting moment due to the main plane is 0·8 Pi', and that due to the tail is 4 Pi', or five times as much. *The tail, therefore, establishes excellent automatic longitudinal stability if it is judiciously disposed.*

It is, moreover, indispensable when the main plane is curved, for then the moment $aPli'$, which arises from the angular displacement of this plane, is an upsetting moment;[1] it is only because the righting moment of the tail is so much greater that the machine remains stable. Therefore:

The tail should be larger, all other things being equal, in an aeroplane with curved planes than in an aeroplane with flat planes.

The degree of longitudinal stability in an aeroplane can be compared with a pendulum. The total righting moment due to the change i' in the angle of incidence is:

$$aPli' + \frac{PmL}{i} i',$$

or

$$Pi' \left(al + \frac{mL}{i} \right).$$

FIG. 58.

If from a point G (Fig. 58) a weight P equal to that of the aeroplane is suspended on an arm of a length equal to $\left(al + \frac{mL}{i} \right)$, and if the arm is moved out of the vertical to the extent of the small angle i', it tends to return to the vertical by the action of a righting moment which is exactly

$$Pi' \left(al + \frac{mL}{i} \right).$$

The degree of longitudinal stability of the aeroplane

[1] Some people suggest that for this reason curved planes should be abandoned for flat ones.

under consideration is therefore the same as if its total weight was applied to the end of the arm of a length as above stated and fastened rigidly to the machine at its centre of gravity.

In the concrete case lately considered, the arm in question would be 4·80 m. in length (of which 4 m. would represent the stabilising efficiency of the tail and 0·80 m. that of the main plane).

The problem of longitudinal stability[1] has just been

[1] It is easy to show experimentally, in a rough and ready but interesting manner, the connection of the results which concern the equilibrium and the automatic longitudinal stability of an aeroplane, the function of the tail and also the glide.

Take a rectangular sheet of paper (which it is best to bend, lengthways, into a dihedral angle, so as to give it transverse stability) and,

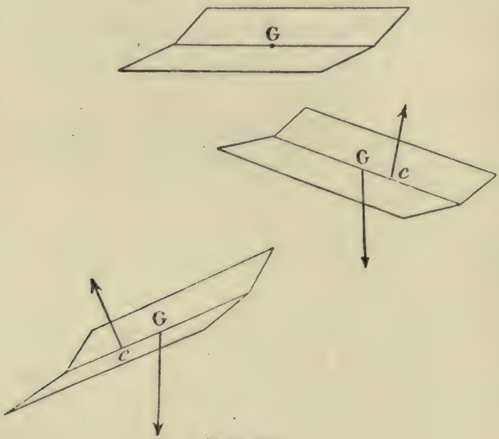

FIG. 58A.

holding it horizontally, allow it to drop. It will fall vertically, keeping its horizontal position. But if from any cause it tilts, it will begin to glide immediately in the direction of the tilt. As the angle of incidence thereby diminishes, the centre of pressure moves forward from G to c (Fig. 58A) and sets up a righting moment, under which the sheet returns to the horizontal, passes it, and takes a tilt in the opposite direction, continuing its fall with similar oscillations, which increase in intensity until it completely overturns.

If this sheet is weighted between the centre and one of the sides with

considered with a non-lifting tail. The same method can be followed with a lifting tail or with a tail struck on its upper surface. With this (see § 52) the longitudinal stability of the aeroplane is excellent. Moreover, as will be seen later on (§ 75), such an arrangement (Fig. 59), which does not seem to have been used hitherto, appears advantageous from the point of view of the effects of the wind; on the other hand, it has a drawback in diminishing the lift of the machine, as the tail suffers a reaction directed towards the ground.

a small heavy object, such as a piece of lead or thick steel wire stuck on by a wafer or gummed paper, it will, on being dropped, tilt and glide, usually with a pitching motion, towards the side so loaded.

By moving the weight forward or by increasing it, the centre of gravity is placed in a spot corresponding to the usual angles of incidence (§ 51). The sheet then takes longer glides, while the oscillations are reduced; for, as has been seen, the smaller the angle of incidence, the more stable is the aeroplane. As the weight, that is, the centre of gravity is placed farther and farther forward, it proves, when a certain position is passed, that it is not possible to maintain equilibrium, and the sheet dives.

FIG. 58B.

By adding a tail (Fig. 58B) in the rear the sheet can find its equilibrium and glide. The angle of glide varies, moreover, with the angle of the tail, and by successive trials the inclination of the tail is found that gives the longest glide. If the

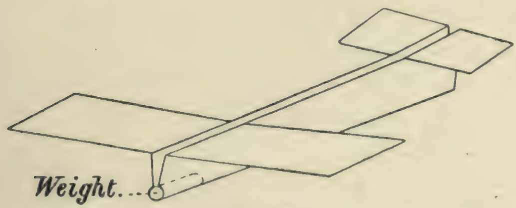

FIG. 58C.

tail is lowered instead of raised, the sheet will dive as before, which proves that this position of the tail, as stated in § 54, gives instability.

Similar experiments can be carried out with a sheet of paper cut out roughly in the form of an aeroplane, such as is illustrated in Fig. 58C.

The necessity of having the tail well away from the centre of gravity increases the weight of the aeroplane through the addition of a fuselage or body, which also increases the detrimental surface. For this reason the correct distance should be observed; a tail too far away from the centre of gravity is most troublesome, especially in a gusty wind (§ 75). (Birds, by the way, with big tails, such as magpies, are not nearly the best flyers.) The tail of the Wright aeroplane is used as an elevator, which greatly diminishes the area of detrimental

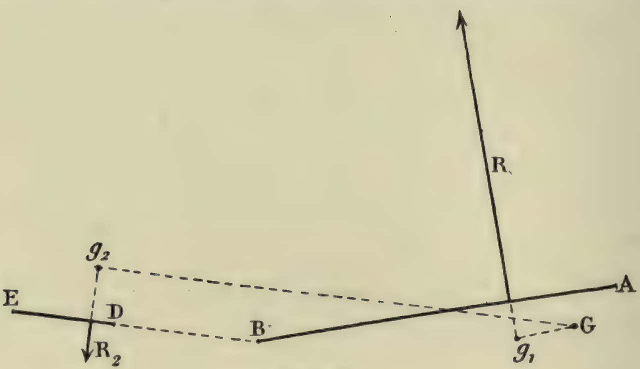

FIG. 59.

surface, and in consequence decreases the head resistance of this machine.

Some remarks may here be made on the influence exerted by the distance between the centre of gravity from the main plane on the longitudinal stability of the aeroplane, which is generally called the *lowering of the centre of gravity.*

Let R_{i_n} (Fig. 60) be the resultant of the air-pressure on a stable aeroplane flying normally. This resultant passes through the centre of gravity, G, of the machine. When the angle of incidence varies as the very small angle i', the resultant moves, and takes up, in relation to its original position one of the three positions numbered

1, 2, 3 on the figure. If the resultant moves parallel to itself (position 1), the righting moment thus created remains constant whatever the position of the centre of gravity on the line R_{i_n} may be. In this case, it can therefore be said that the lowering of the centre of gravity does not exert any influence on the longitudinal stability. If the resultant takes up the position 2, cutting the line R_{i_n} above the centre of gravity, the lowering of this point, from G to G′ for example, increases the righting moment and therefore the stability.

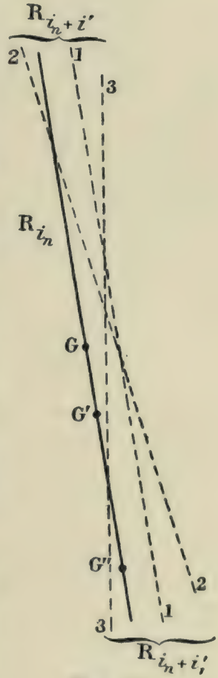

FIG. 60.

Again, in position 3, the lowering of the centre of gravity diminishes the stability and can change it into instability, if it descends below the point of intersection of the allied resultants, to G″ for instance. On the other hand, if the aeroplane be unstable, the lowering of the centre of gravity will diminish its instability, and will be capable of changing it into stability.

Let us apply these geometrical considerations in practice. In an aeroplane consisting of a single flat plane, the allied resultants are nearly parallel; though they try to assume position 2, as can be seen by reference to the results of M. Eiffel's experiments.[1] The lowering of the centre of gravity in such an aeroplane exercises an insignificant influence, though in the direction of increasing stability.

According to the same experiments, when an aeroplane of a single curved plane is used, the allied resultants take

[1] *Mémoires de la Société des Ingénieurs civils de France* (Bulletin de janvier, 1910).

up position 3, but we know in this case that the aeroplane is unstable. The lowering of the centre of gravity will diminish its instability and could, if low enough, make the machine stable.

Both these cases are, as has been said already, quite hypothetical, the aeroplane with one plane being imaginary. With the usual tailed aeroplane the allied resultants always[1] assume position 2, but their point of intersection is generally high above the planes. Therefore:

The lowering of the centre of gravity only increases the longitudinal stability of the aeroplane in quite an insignificant way.

This conclusion is contrary to a widespread opinion which is founded on the error, mentioned at the beginning of the present chapter, which compares the equilibrium of the aeroplane with that of a floating body.

55. The function of the tail in damping out oscillations.

When the equilibrium of an aeroplane which possesses good automatic longitudinal stability is disturbed, a righting moment is set up which restores it. It does not stop, however, at the position of equilibrium, but under the impulse received, passes it and sets up a new righting moment in the opposite direction to the first. Thus a series of oscillations or pitching motions arise that must be damped out as quickly as possible.

First of all, to get this damping effect, the oscillations must not be allowed to assume so great proportions that the righting moment of the tail is insufficient, so that the machine overturns, but the motion resulting from the oscillations must be kept smaller than the moment of

[1] Position 3 cannot be realised with a tailed aeroplane, unless the main plane and the tail form an inverted dihedral angle, and this will occasion instability, except in very special and entirely theoretical cases.

inertia of the machine around the axis, perpendicular to the plane of symmetry, passing through the centre of gravity (one of the three principal axes of inertia). Therefore :

It is essential that the stabilising efficiency of the tail should be proportional to the value of the moment of inertia of the aeroplane in a longitudinal direction.

A machine heavily weighted fore and aft, that is to say, with a large moment of inertia, and having only a small tail, sets up a rocking motion which speedily culminates in a complete upset.

We may conclude, then, that a condition of stability exists defined by the necessary relation between the longitudinal moment of inertia and the stabilising efficiency

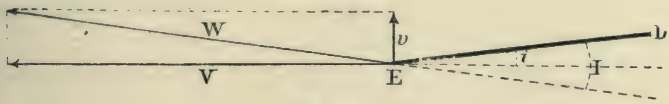

FIG. 61.

of the tail measured by the product of its surface and its lever-arm.

The function of the tail in damping out the longitudinal oscillations of the aeroplane is not confined to the creation of a righting moment in any *static* sense which has hitherto been considered. Should these oscillations have an appreciable angular velocity, they will endow the tail with a vertical speed which is important because it creates a *dynamic* righting moment, thus damping out the pitching.

Let DE (Fig. 61) be a tail, in a position of equilibrium, struck by the wind at the angle i. If, by oscillating, it descends at the speed v, the air strikes it at the same speed v in an upward direction. This speed is compounded with the speed V of the air-flow occasioned by the passage of the aeroplane. Let W be the resultant speed. The tail is therefore, when oscillating, subject to the action of a

current of air of speed W, which is greater than V, striking it at an angle I, which is greater than i.

In addition to the static righting moment arising simply from the inclination of the machine, a dynamic righting moment is produced, the importance of which increases with the rapidity of the oscillation. The increase in the angle of incidence having a value, in Fig. 61, of $\frac{v}{V}$, the excess pressure exerted on the tail is $KsV^2 \times \frac{v}{V}$ or $KsVv$. The dynamic righting moment is only proportional to the speed of the aeroplane while the static righting moment is proportional to the square of this speed. The dynamic righting moment is also proportional to the linear speed v of the tail, which increases with the length of the lever-arm and with the rapidity of oscillation. To sum up:

The tail acts as a damper-out of oscillations by creating a dynamic righting moment.

A final pitching effect has to be considered. When the angle of incidence diminishes as a result of oscillation, since the motive power remains constant, the aeroplane descends, and similarly as the angle increases it rises. Consequently the trajectory of the centre of gravity would be a series of curves. But, owing to the inertia, the action of this diminution in the lift affects the trajectory less as the speed increases, and if oscillation is sufficiently rapid, the lift can recover and surpass its normal value before the trajectory is appreciably varied.

For the reasons just mentioned, it will be advantageous to diminish the moment of inertia by concentrating the weight near the centre of gravity, so as to make the oscillations as rapid as possible. (By virtue of this, lowering the centre of gravity which increases the moment of inertia becomes detrimental if carried to any extent.) On the other hand, pitching oscillations should be relatively

slow, so that the aviator can anticipate them and correct them with the elevator.

Moreover, the aeroplane is less sensitive to disturbing influence (see Part III.) as its moment of inertia increases. A middle course must, therefore, be steered between these two contradictory conclusions, and, as M. Soreau writes, "aeroplanes judiciously loaded fore and aft are particularly stable, and have a happy dislike of oscillating. But they must be fitted with more powerful tails." In practice, however, constructors are disposed to concentrate the weight rather than to distribute it.

III.—Lateral Stability

56. The lateral equilibrium of an aeroplane on its flight-path.

At first sight it seems that symmetrical construction insures lateral equilibrium. This is quite true if a

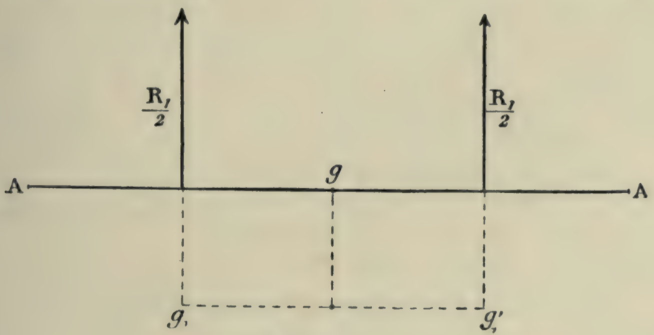

Fig. 62.

machine is imagined consisting of a straight plane, that is, of two wings perpendicular to the place of symmetry placed in the same straight line. As the wings are of equal dimensions, the air pressure upon each is identical and at an equal distance from the centre of gravity, so that equilibrium necessarily follows (Fig. 62). This reason-

ing is applicable whether the plane of symmetry is vertical or not, so that *the transverse stability of an aeroplane, supposed to consist simply of a straight plane, will be indifferent.*

57. The keelplane.

To reduce an aeroplane to a single plane is, of course, a theoretical fiction; but in practice one must take into consideration, in dealing with transverse stability, the idea of a *keelplane.*

When an aeroplane is struck laterally by a current of air perpendicular to its plane of symmetry, the components representing the action of the air on the various parts of the machine in the direction of the current result in a force applied to a certain point C_Q.

By an assumption similar to the ideas of the detrimental surface (§ 11) and of the imaginary tail (§ 52), it may be taken that all parts of the aeroplane offering lateral resistance can be replaced by a single hypothetical surface of a size S_Q, called the *keelplane,* the centre of which is the point C_Q.

The keelplane is most pronounced in certain aeroplanes with vertical partitions, such as the early Voisin, or in machines whose planes are set at a transverse dihedral angle. On the other hand, it is reduced to a minimum in such machines as that of the Wright Brothers, but it exists even in them. In fact:

All aeroplanes have keelplanes, in the same way as they have a detrimental surface and an imaginary tail.

When an aeroplane, flying horizontally, is struck laterally by a horizontal gust of wind, which instead of being perpendicular to the plane of symmetry is at an angle to it, the action of the air on the various parts of the machine will in each case have a component perpendicular to the said plane of symmetry.

If all these forces are taken, their resultant passes

through the plane of symmetry and the keelplane at a certain point C_q, which can be called *the centre of pressure of the keelplane.*

When the direction of the horizontal lateral air-current alters, that is to say, when the angle at which it strikes the plane of symmetry of the aeroplane moves from 90° to 0°, the centre of pressure of the keelplane moves from the position C_Q, its centre, to a limit point C_0, which is reached when the air-current strikes the machine head-on.

The study of lateral stability and of the wind leads, as will be seen later on, to a consideration of the effects of lateral air-currents striking the keelplane of the aeroplane at small angles. The centre of pressure C_q, corresponding to the action of such currents, remains near the limit-point C_0, just defined, the position of which partakes, therefore, of a special interest.

58. The axis of lateral rotation.

The keelplane, as we have seen, plays an important part in the lateral equilibrium of the aeroplane, but there is another factor which must be taken in account, and that is the position of the axis of rotation around which the lateral oscillations occur. This line is the same as the principal longitudinal axis of inertia of the machine.

It can be considered as being rigidly fixed to the aeroplane, and an integral part of it. Briefly, this axis of rotation passes through the centre of gravity and is contained in the keelplane, which coincides with the plane of symmetry of the aeroplane.

59. As a rule, the aeroplane has only one position of lateral equilibrium.

If the axis of rotation were coincident with the trajectory of the aeroplane's centre of gravity, this trajectory would remain in the plane of the keel when the machine heels, since the rotation takes place around it. Thus the

lateral inclination of the aeroplane would not alter the keel-plane's angle to the air-current, or by this means set up any righting or upsetting moment; the equilibrium would remain indifferent. But this is an exceptional case, and usually the axis of rotation, which as we have just seen is a fixed quantity, does not coincide with the trajectory of the centre of gravity, which depends on the angle of incidence of the main plane.

If Fig. 63 is considered as a section of the aeroplane perpendicular to the axis of rotation, and if AA′, I, t, Is, are

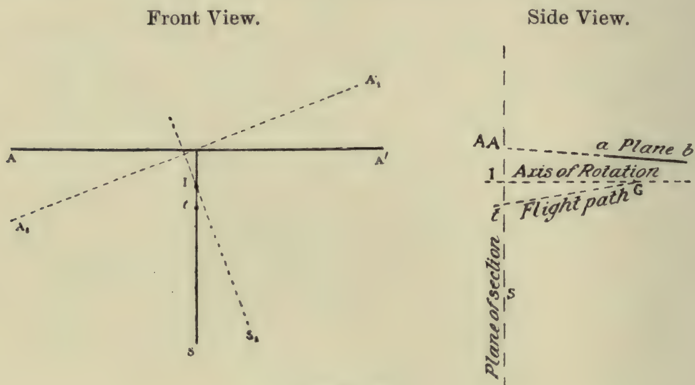

FIG. 63.

taken as the respective paths of the main plane, of the axis of rotation, the trajectory of the centre of gravity, and the keelplane (or plane of symmetry), when the plane is inclined as $A_1A'_1$ the points I and t do not alter, and the path of the keelplane becomes Is_1. The trajectory of the centre of gravity remaining on the side t of Is_1, a reaction of the air is produced by the inclination of the aeroplane on this side of the keelplane, which is proportional to the square of the speed.

If the point where this force goes through the plane of symmetry, that is, if the centre of pressure, C_q, of the keel-plane (which, being struck at a small angle, is near its

limit point C_0 (§ 57)), moves *above* the axis of rotation, the reaction caused sets up a righting moment, if *below*, an an upsetting moment.

It is only when the point C_q coincides with the axis of rotation—a rare event—that no moment is created, and the equilibrium remains indifferent.

In every way it is clear that—save in exceptional cases where the trajectory of the centre of gravity coincides with the principal longitudinal axis of inertia or with the axis of rotation, or where the centre of pressure of the keel is upon this axis—

There is only one position of lateral equilibrium of the machine on its flight-path, and that is when the plane of symmetry is vertical.

60. Automatic lateral stability.

It has just been explained that the lateral equilibrium of the aeroplane—attainable only in one position—can be stable or unstable as the axis of rotation and centre of pressure of the keelplane assume various positions relatively to each other. Therefore:

An aeroplane may possess automatic lateral stability just as it may possess automatic longitudinal stability.

The keelplane [1] acts as the agent in the former in the same way as the tail acts in the latter.

To discuss the influence of the relative positions of the plane, the axis of rotation, the keelplane, and the trajec-

[1] Apart from the action of the keelplane, either a righting or upsetting moment can be created, from the fact that when the axis of rotation does not coincide with the trajectory of the centre of gravity, the forward edge of the plane does not remain horizontal but heels over. If the lower side moves in advance of the other, a slight displacement of the centre of pressure is caused towards this side, and, in consequence, a righting moment; while if the higher side moves in advance of the other, the moment caused is an upsetting one. In any case the effect is of little importance.

tory of the centre of gravity would take too long. It can be briefly summed up as follows:

Firstly, to obtain automatic lateral stability, it may be necessary to make the axis of rotation coincide as nearly as possible with a line drawn parallel to the plane of the wings through the centre of gravity.

Secondly, the limit point C_0 of the keelplane's centre of pressure ought to be placed above the axis of rotation.

In these circumstances automatic lateral stability increases with the angle of incidence of the plane. It would reach its maximum in the case of a vertical gliding descent of a horizontal plane, if such were possible.[1] (See § 40.)

On the contrary, it diminishes and may even disappear when the angle of incidence becomes very small.

Similar considerations have led to the adoption in some machines of planes set at a transverse dihedral angle or V, or of a vertical keel above the plane[2] (Fig. 63A). But as this arrangement presents serious disadvantages when the

[1] It is not possible, because in the case of the aeroplane longitudinal equilibrium at an angle of incidence in the neighbourhood of 90° is either impossible or unstable. (See §§ 52 and 54.)

[2] It is generally admitted that a dihedral plane ACA′ (Fig. 63A) is equivalent to a plane *aa′* furnished with a vertical keel, CD. From the standpoint of lateral stability, however, this is not justifiable, for the

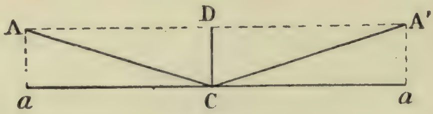

FIG. 63A.

dihedral plane sets up a greater righting moment when tilted at a similar angle than the keel. In fact, it will be shown (see footnote, p. 148, and footnote, p. 151), that if the axis of rotation is brought sufficiently near to the apex of the dihedral angle, the result of lowering one wing is to increase its angle of incidence on this side, while that of the raised wing is decreased. Two united righting moments are the result, and their lever-arm, equal to half the span of a wing, is considerable. On the other hand, the lever-arm of the single moment of the keel CD is quite small.

machine is struck laterally by a wind gust (§ 77), it is gradually being abandoned.

The inverted dihedral form or $\wedge$ (Fig. 64) is usually supposed only to effect an unstable equilibrium, and this is so when the angle of incidence has a certain value, especially in the case of a vertical parachuting descent (which is impossible, as has been shown).

But it is capable of conducing to stable equilibrium—*and the more so as the angle of incidence decreases*—when the axis of rotation and the plane make between them an

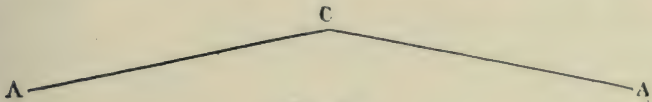

FIG. 64.

angle greater than the said angle of incidence (in this case the point I (Fig. 63) will be below the point t): this is perhaps the explanation of certain sea-birds placing their wings in this position.[1] The effect of lateral gusts is much less to be feared with the inverted dihedral form than with the other (see § 77).

The question can be summed up as follows:

The lowering of the centre of gravity is usually only of quite small importance from the standpoint of automatic lateral stability.

However, the change in position of the axis of rotation, which results therefrom, is capable of influencing in a certain degree the lateral automatic stability which is peculiar to certain plane-shapes and to certain arrangements of the keelplane (perhaps to the extent even of rendering stable laterally an unstable aeroplane, or conversely).

It must be noted that a lateral inclination of an aero-

[1] It is not impossible, as will be shown later (2nd footnote to § 77), that, in the future, when the head resistance of the aeroplane is lessened and it can fly at small angles of incidence, some advantage may be gained by placing the wings at a slight inverted dihedral angle.

plane usually leads to a longitudinal inclination of the machine:

Rolling, in fact, produces pitching.

That follows from reasons that may now be stated.

Firstly, according as the angle formed by the axis of rotation and the main plane is greater or smaller than the angle of incidence (that is, as, in Fig. 63, the point I is above or below the point *t*), so the tilt of the plane increases or diminishes the value of the said angle of incidence, which can be proved geometrically.[1]

Secondly, the analogous effect produced on the tail is, usually, different, and sometimes even inverse, according as the axis of rotation, projected, passes above or below it.

Therefore, the result of a lateral tilt is generally to disturb the longitudinal equilibrium and, consequently,

[1] By redrawing Fig. 63 (Fig. 64A), it is seen that the line $A_1A'_1$ being tangential to the circle described, with I as the centre and Ia as radius,

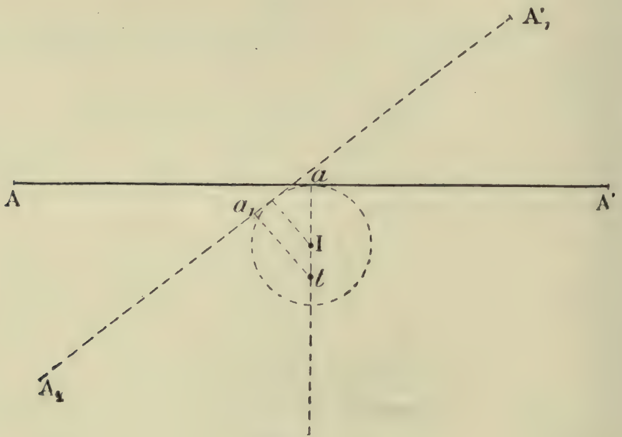

FIG. 64A.

the distance ta_1 is smaller than ta, which proves that, in the position shown in the figure, the angle of incidence is smaller when it is tilted than when it is not.

to cause a pitching motion, save only if the machine has been built so as to prevent this.

The position of the tail with respect to the axis of rotation is thus of considerable importance.

61. The function of the keelplane and of the main plane in damping out oscillations.

An aeroplane, laterally stable, on being disturbed from its position of equilibrium, is subject to a righting moment which restores it and throws it back beyond this position. Thus rolling oscillations are set up in the same way as pitching oscillations. It will be obviously desirable that the impulse arising from the effect of these oscillations should not become of such importance that the righting moment cannot cope with it, and one conceives that there ought to exist, as in the case of longitudinal equilibrium (§ 55), a condition of stability determined by a relation between the lateral moment of inertia of the aeroplane and the forces which compose the stabilising efficiency of the keelplane. But, as will be seen a little later (§ 63), the use of methods of dynamic stabilisation operated by the aviator, neutralises the devices for automatically insuring the lateral stability of the aeroplane. Therefore, the consideration of this condition of lateral stability is usually neglected.

On the other hand:

The main plane, in damping out the lateral oscillations of the aeroplane, plays a part similar to that of the tail, which damps out the longitudinal oscillations.

Each wing acts in the same way as the tail, and a dynamic righting moment arises from the rolling motion, which is proportional to the rapidity of the oscillations, to the speed of the aeroplane, and to its span. This action tends to stop transverse oscillation, but only when the oscillation is sufficiently rapid; *it cannot of itself prevent the aeroplane from slightly slipping sideways under the*

influence of various small disturbing forces; such motion can only be stopped by the creation of a righting moment arising either from the existence of lateral automatic stability or by the direct manipulation of the aviator. From both these points of view it is profitable to reduce the lateral moment of inertia of the aeroplane, that is to say, to concentrate the weight towards the plane of symmetry, but, as has been said *à propos* of longitudinal stability, it is best that the oscillations should be slow in order that the aviator can see them coming and remedy them in time. Just as in the matter of longitudinal stability, there is a middle course which may be taken between these two contradictory conclusions.

62. The effects of skidding.

Lateral oscillations can have the effect, especially when they are slow, of causing the plane to slide laterally, that is to say, to skid each time it tilts. If the normal speed of the aeroplane is small, the result is a zigzag trajectory of the centre of gravity, a movement which combined with the oscillation itself produces a balancing effect.[1]

This effect disappears when the keelplane has a certain value, thus causing it to act as a damper-out of oscillations, and, if in addition its centre of pressure is above the axis of rotation, as a righting element.

63. Dynamic lateral stability effected by the aviator.

The damping effect noted in § 61 *is capable of checking the rocking motion of the aeroplane but not of preventing it.* On the other hand, the automatic lateral stability which an aeroplane can possess is not comparable in importance with the automatic longitudinal stability insured

[1] This can be observed with a sheet of paper weighted in the way shown in the footnote on page 134, but without the dihedral angle (the paper must either be thick or the sheet must be small).

by the tail.[1] Besides, the extension and raising of the keelplane, which usually increase lateral stability, present serious disadvantages in the event of lateral gusts. For

[1] Although the study of such a question leads to more advanced mathematics and to the use, hitherto avoided, of trigonometry, it seems necessary to make clear, by an example, the importance of the transverse righting moment in the particular case of a dihedral plane with the apex parallel to the axis of rotation.

Let p' (Fig. 64B) be the right wing of the plane, making with the

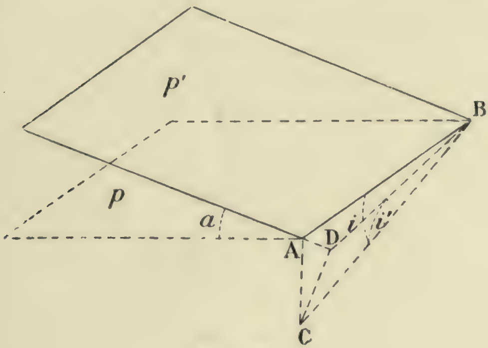

FIG. 64B.

plane p drawn perpendicularly to the plane of symmetry from the apex AB the angle a. On the other hand, let CB be a line drawn from B parallel to the trajectory of the centre of gravity. (For the sake of clearness in perspective this trajectory has been taken in the figure as descending.)

The angle of incidence of the plane p will be $CBA = i$; that of the wing p' will be $CBD = i'$; the point D being the projection of C on the plane p'. Taking the angles i and i' as small, it is easy to see the relation between them, that is, $i' = i \cos a$.

If the plane were not dihedrally bent, the action of the air on the wing p would be

$$F = K \frac{S}{2} V^2 i,$$

S representing the total wing surface. On p', supposing it equal to p, it would be

$$F' = K \frac{S}{2} V^2 i'.$$

Therefore

$$F' = F \cos a.$$

this reason, in certain types of machines little regard has been paid to the development of automatic stability. In some, the Wright aeroplane for instance, automatic stability

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When the angle a varies as a', the difference f' of the force F′ is

$$f' = \mathrm{F}\,[\cos a - \cos (a + a')].$$

That is, $f' = \mathrm{F}a' \sin a$, taking, as a' is small, $\sin a'$ to equal a' and $\cos a'$ to equal 1.

In Fig. 64C, l and g represent the span of the wing and the height of

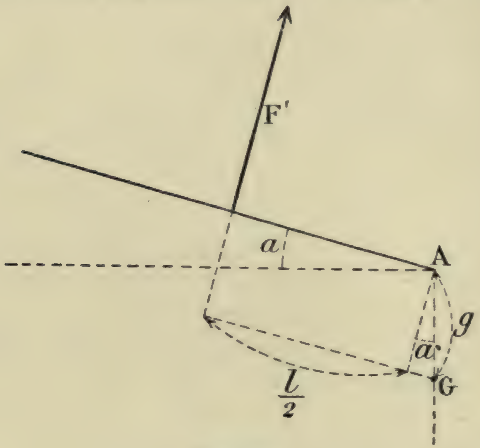

FIG. 64C.

the apex above the axis of rotation (the two lines are supposed to be parallel), then the moment of the force f' is expressed as far as it concerns the wing by

$$f'\left(\frac{l}{2} + g \sin a\right).$$

The moment of the other wing, which is equal to the above, is added to this, and, finally, the total righting moment is represented by

$$\mathrm{M} = 2\mathrm{F}a' \sin a \left(\frac{l}{2} + g \sin a\right),$$

Or substituting its value for F,

$$\mathrm{M} = \mathrm{KSV}^2 i\, a' \sin a \left(\frac{l}{2} + g \sin a\right).$$

But the weight P equals the sum of the vertical components of the two forces F′, so that one can write:

$$\mathrm{P} = \mathrm{KSV}^2 i \cos^2 a.$$

is discarded, or nearly so, in favour of indifferent lateral equilibrium, which is advantageous in view of lateral gusts.

It has been preferred in these machines to put at the aviator's disposal a means for producing at will, in the event of an involuntary inclination of the plane, a righting moment proportional to the square of the speed and operating at a considerable distance from the centre of gravity, which insures a very strong action. The method usually consists in increasing the angle of incidence at the end of the lowered wing, and of decreasing it at the end of the raised wing, either by warping, as is done by the Wright Bros., who invented this method, or by the use of balancers or small planes moving about an axis parallel to the forward edge of the main plane.[1]

So the righting moment takes the following definite expression :

$$M=\frac{P}{\cos^2 a}\sin a\left(\frac{l}{2}+g\sin a\right)a'.$$

In § 54 the longitudinal stability of an aeroplane was compared to that of a pendulum of equal weight ; if we compare results, the length of the pendulum spindle should be expressed by:

$$\frac{\sin a}{\cos^2 a}\left(\frac{l}{2}+g\sin a\right).$$

The transverse stability, therefore, increases as the angle a, that is to say, as the dihedral angle is more pronounced.

For the purpose of estimating the degree of stability that it gives, let us suppose, for example, that $\sin a=0{\cdot}2$ (so that the angle a is 12°, and that its cosine differs but little from unity), and that $l=5$ metres and $g=0{\cdot}50$ metres.

The length of the pendulum spindle will then have a value of $0{\cdot}2\ (2{\cdot}5+0{\cdot}5\times0{\cdot}2)$, say, 0·52.

This proves fairly conclusively that automatic lateral stability is much less than the automatic longitudinal stability produced by the tail, since, taking the numerical example given in § 54, the length of the spindle works out at 4·80 metres.

It will also be seen how lowering the centre of gravity has quite an insignificant effect on the lateral stability of an aeroplane, for even lowering g, twice as far, to 1 m., the length of the pendulum spindle is only increased by 2 centimetres.

[1] Other ways of dynamic stabilisation can be imagined, for instance, by increasing the span of the lowered plane by means of a small sliding panel which would prolong it. This arrangement could perhaps be made into

In the first Voisin machines no such arrangement existed. It is claimed that these machines possessed the utmost automatic lateral stability from the fact of their large spread of keelplane, the centre of which, moreover, was raised considerably above the axis of rotation.

In addition, when the automatic stability was not sufficient to prevent the machine tilting, the aviator could re-establish equilibrium by a suitable movement of his vertical rudder.

IV.—Directional Stability

64. The aeroplane ought to fly head to wind.

The aeroplane ought to be constructed in such a way that if, as has been hitherto supposed in the consideration of questions of equilibrium and stability, it be suspended from its centre of gravity and subjected to a wind current equal to that created by its speed, its plane of symmetry would adjust itself so as to lie directly in the wind's eye. In other words:

The aeroplane ought always to fly head to wind,

and should not turn aside or veer with a small disturbing influence. In fact it ought to behave *like a good weather-cock*, and in this sense also "*lie on its trajectory.*"

The problem of the weathercock is quite simple. If the point G (Fig. 65) is the projection of the axis of the plane of the weathercock AB, and if this point is behind the axis of the limit point C_o of the centre of pressure, the plane AB takes up a position A′B′ under the action of wind blowing in its direction, thus making with its original position the angle i, so that the centre of pressure C_q is on the axis G.

A weathercock like the one described will be, moreover,

a means for establishing dynamic automatic stability if there were two of these little panels rigidly connected, thus forming a moving apparatus capable of sliding very freely under the effect of gravity, at the slightest transverse inclination of the aeroplane.

a bad one, as it will be always several points out from the true direction of the wind. In order that it should act correctly, the axis G ought to be in front of the limit point C_0 of the centre of pressure. The weathercock will then lie head to wind, and be the more stable in proportion as these two points are farther apart.

In aeroplanes this result is obtained if the centre of gravity is in front of a certain point C_0 of the keelplane, which is the same as the limit point of the centre of pressure of the said plane (§ 57).

Bearing this in mind, the centre C_Q of the keelplane must be placed considerably behind the centre of gravity of the aeroplane.

The boat-shaped form given to the body in certain machines is usually enough to secure this, but, at the same time, the bow must not be too big, or the machine will deflect.

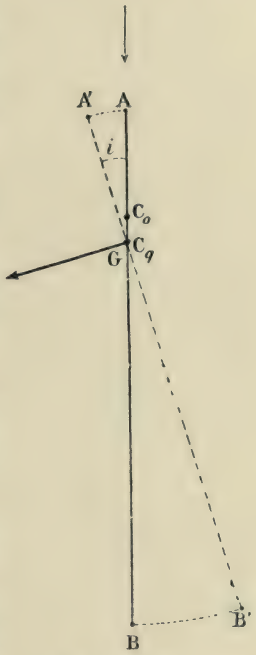

FIG. 65.—Plan.

In other machines it is necessary to place a vertical keel in rear, which usually includes a movable part constituting the *rudder* (§ 65).

CHAPTER V

TURNING

65. The action of the vertical rudder.

The first idea that comes to mind when one desires to furnish an aeroplane with an organ, whose operation will cause it to change the direction of its flight in a horizontal sense, is to give it a vertical rudder in the rear such as is used in boats and dirigible balloons, but in order that such an organ can fulfil this function, the aeroplane must have a certain resistance to lateral motion, that is to say, an adequate keelplane.[1]

If one takes the imaginary case of an aeroplane with no keelplane at all (Fig. 66), and consisting of a plane AA′ BB′ and a vertical rudder CD, when this rudder is placed in the position CD′ it experiences, on account of the speed, a perpendicular reaction p, which will have the effect of turning the machine round its centre of gravity until the rudder again lies head to the wind.

The aeroplane will take up in consequence a direction oblique to the wind. In other words, it will be deflected, *but will, notwithstanding, continue to advance in a straight line,* the only horizontal forces which affect it, the thrust t and the head resistance r being equal and opposite to each other along the axis of the machine.

The desired end is, therefore, not attained, and one can see that

The operation of a vertical rudder on an aeroplane

[1] It is for this reason that keeled boats obey their helm better than flat-bottomed boats.

without a keelplane, if such a machine were possible, only produces a deflection of the machine, without altering its original trajectory.

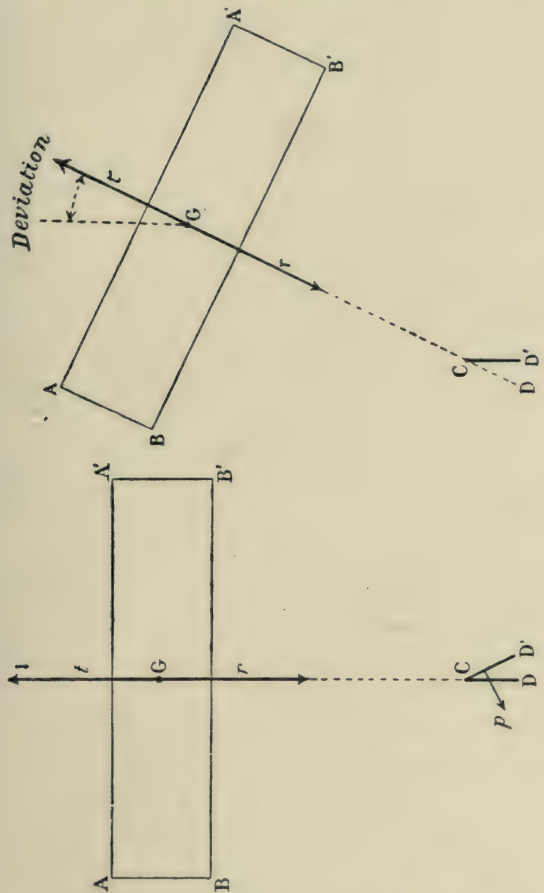

FIG. 66.—Plan.

If it be supposed, however, that there is a keelplane EF (Fig. 67), so that the aeroplane has directional stability, the machine will assume on its flight-path, when the horizontal rudder is moved from CD to CD′, a position of equilibrium, in which the pressures p and q exerted in

opposite directions by the air on the vertical rudder and on the keelplane have equal moments with respect to the centre of gravity. (This problem of equilibrium is similar to that which has been studied in § 54, Fig. 59.)

But it must be taken into consideration that the forces q and p, being unequal, have a resultant equal to $q-p$,

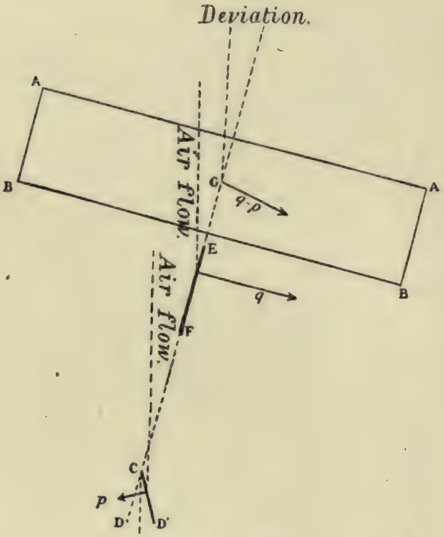

FIG. 67.—Plan.

which passes through the centre of gravity, since both forces balance themselves around this point. The centre of gravity is subject, therefore, to the continuous centripetal action of the forces $q-p$, which tends to curve the trajectory until this curvature sets up a centrifugal reaction, which balances the forces $q-p$.

In this case the vertical rudder operates effectively and turns the aeroplane.

The turning action is the more pronounced and the radius of the turn is shorter, as the pressure q is larger and the pressure p is smaller. The value of the pressure

q increases with the effective surface of the keelplane and with the size of the angle of deflection. Fig. 67 plainly shows that the latter is always smaller than the angle through which the rudder is turned. Moreover, it more nearly approaches this angle as the centre of pressure of the keelplane approaches the centre of gravity. If these two points coincide, the angle of deflection will be absolutely equal to the angle through which the vertical rudder is turned. (This is a similar case to that of the aeroplane with a non-lifting tail. See § 54, Fig. 57.) On the other hand, the pressure p decreases with the area of the rudder.

To sum up:

In order that the action of the vertical rudder may be efficient, it is necessary to increase the effective surface of the keelplane and to bring it near the centre of gravity, so that it can maintain directional stability and, at the same time, to reduce the area of the vertical rudder. Consequently the rudder is placed at a considerable distance from the centre of gravity, in order that it may possess an adequate lever-arm.

This explains the part played by the vertical partitions of a biplane, such as the original Voisin machines, which formed a considerable part of the keelplane, and were in front of the centre of gravity. The dihedral form given to the wings in other machines has a similar effect besides that of stability. Another result of turning the keeled aeroplane ought to be mentioned. The centripetal force $q-p$ has a component in the direction of the trajectory which increases the head resistance, so that the aeroplane drops as it turns, unless the aviator increases the angle of incidence, and consequently the lift, by means of his elevator.

Such increase in the angle of incidence is produced automatically if the rudder and the keelplane are high enough. The component previously mentioned then passes above the centre of gravity and produces a moment which increases the angle.

66. Result of tilting the planes.

As the keelplane in certain machines is reduced to a minimum, the action of the vertical rudder is insufficient to enable a turn to be made in a small radius. Other means are, therefore, brought into play.

When an aeroplane turns, the extremities of the wings move at different speeds, with the result that the outside

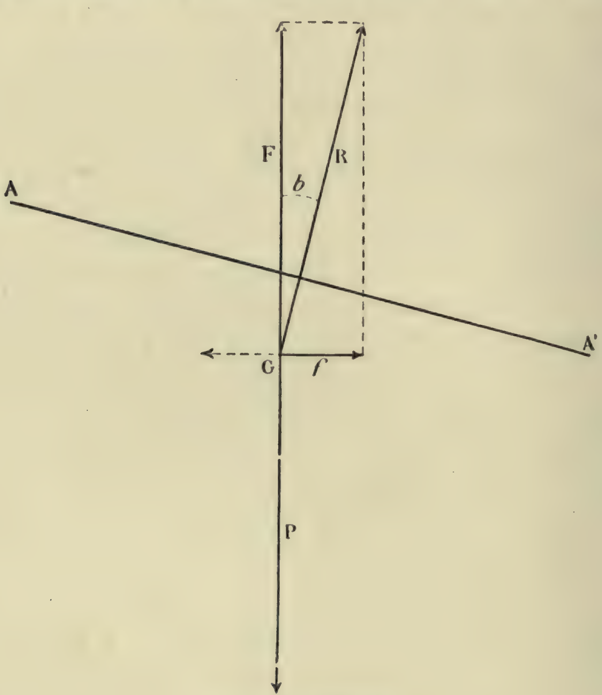

FIG. 68.—Elevation.

wing rises and the inside wing, towards the middle of the circle described by the machine, is depressed. The aeroplane, in fact, heels.

The pressure R of the air (Fig. 68) on the plane inclines with it, taking up a lateral component f, which always, acting through the centre of gravity, produces a centripetal effect.

If the turn results from the action of the rudder, the force f adds itself to the resultant $q-p$ (§ 65), and the sum of these two forces should be equalised by the centrifugal reaction.

The lateral inclination which is inevitable may also be employed as a cause as well as an effect. If by any means the machine is inclined, the centripetal component of air-pressure immediately induces a turn without the aid of the rudder, and this is done when the keelplane is too small to insure the effective operation of the rudder.

The most practical way of doing this is by warping or by similarly acting arrangements, such as balancers, whose primary use is to insure lateral dynamic stability (§ 63).

The use of these methods has a secondary effect of increasing the head resistance on the side where the angle of incidence is increased, which tends to turn the aeroplane in the reverse direction to that desired. To prevent this occurring, an antagonistic moment must be created by the rudder.

This is the chief use of the rudder in the Wright machine, which can be used separately or conjointly with the warping.

67. Relation between the radius of the turn and the speed of the aeroplane.

Referring to Fig. 68, in the case of an aeroplane without a keelplane, that is to say, when the horizontal component f of the air pressure alone balances the centrifugal force, the inclination b, which represents the tilt of the machine, is equal to the force f divided by the vertical component F of the pressure, and as F equals the weight P of the aeroplane:

$$b=\frac{f}{\mathrm{P}}.$$

And as the centrifugal force $f=\frac{\mathrm{PV}^2}{gr}$ (g represents the

acceleration 9·8, or in round numbers 10, of gravity), the value of the tilt of the aeroplane turning in the radius r at the speed V is given by the formula :

$$b = \frac{V^2}{10r}. \qquad (25)$$

The tilt is therefore proportional to the square of the speed, and inversely to the radius of the turn.

As a certain lateral inclination should not be exceeded, it can be laid down that *sharp turns should not be made at a high rate of travel.* For example, if 0·30 m. per metre is fixed as the admissible maximum tilt, turns must not be made at a speed of 15 m.p.s. in a radius less than 70 m.; at a speed of 20 m.p.s. in a radius of less than 130 m., and so on.

Too much importance, however, should not be attached to these figures, which are quite theoretical, as they relate to a hypothetical instance of an aeroplane without a keel-plane.

In a well-constructed aeroplane the tilt will be less than indicated in formula (25), and consequently the limit of the radius corresponding to the turn made at a certain tilt will be less than has been said; the more so, in fact, as the size of the keelplane increases and as its position is further in advance relatively to the centre of gravity.

68. Lateral equilibrium in turning.

We may sum up the foregoing statements by saying, that the centripetal force which occasions the turn of an aeroplane can be due:

1. To the pressure of the air on the keelplane and on the vertical rudder set up by the operation of the latter; or
2. By the tilting of the plane resulting, by the wings having different speeds, from the raising of the wing exterior to the turn.

We will take the former case first.

If the centre of pressure of the keelplane is above the axis of rotation, as is usual in aeroplanes having automatic lateral stability, the reaction on this point sets up a moment which tries to tilt the machine towards the inside of the turn.

The reaction on the vertical rudder sets up an inverse moment, but in the case under consideration it is less than the former, since—the rudder being taken as effective—the pressure it experiences is less than that on the keelplane. (An instance where it is not so will be considered later on.)

At the same time the unequal speed of the wings produces, from the increased pressure on one of them, another moment also tending to tilt the aeroplane inwards. Under this double action the tilt increases constantly, and, by the cause and effect changing places, diminishes the radius of the turn more and more, finally causing the machine to fall if not checked by an antagonistic moment, which in aeroplanes with good automatic lateral stability is set up by the inclination of the machine.[1]

Thus the aeroplane takes up on its curved trajectory an inclined but stable position, so that at the same time the righting and upsetting moments as well as the centripetal and centrifugal forces are equal.

As the centripetal force is the sum of the pressure on the keelplane and vertical rudder and the horizontal component of the inclined plane, it is clear that the tilt necessary to produce a centrifugal force sufficient to balance it will be less when the machine has a keelplane of an appreciable size near the centre of gravity, than indicated in formula (25) which is applicable to an aeroplane without a keelplane.

Taking the second case, mentioned at the beginning of

[1] Equality between the two moments is not arrived at without creating reactions. Therefore, aeroplanes with large keelplanes turn abruptly, and there is a risk of straining them in trying to turn too quickly.

this section, of an aeroplane with an inefficient keelplane which is made to turn through the raising of one wing, it will be seen that the upsetting moment thus caused has not any antagonistic moment to balance it. It follows that any turn will result in a fall, if the aviator does not intervene and create, by a contrary action, an inverse moment, so that equilibrium in the tilted position suitable to the turn desired may be possible.

Formula (25) is applicable to the present case in which the lateral component of pressure alone balances the centrifugal force.

The tilt taken by aeroplanes with inefficient keelplanes, such as the original Wright, in order to turn in a certain radius at a certain speed, is bigger than that taken under similar conditions by aeroplanes with efficient keelplanes, such as the original Voisin.

It has been assumed above that the moment exerted around the axis of rotation during a turn, by the action of the air on the vertical rudder, was smaller than that on the keelplane, and it has been seen that this inequality tends to tilt the aeroplane inwards on the turn.

When this tendency is considered too great, it can readily be reduced by raising the position of the rudder. By this means the righting moment that it sets up is increased. *The rudder must not, however, be raised so high as to produce a righting moment greater than the upsetting moment on the keelplane, or the effect will be to turn the machine in the reverse direction to the one desired.*

69. Loss of elevation due to turning.

In § 65 we saw that in aeroplanes with considerable keelplanes the deflection made through the rudder resulted in an increase of head resistance, and consequently caused a loss of elevation. Apart from this special case, *an aeroplane generally falls while turning,* as the tilting of the plane reduces the horizontal component of air pressure

upon it, and consequently the lift becomes less than the weight (Fig. 69).

This effect increases with the tilt, and it is obvious that it is the more pronounced in the case of aeroplanes with an inconsiderable keelplane. *Turning at a small elevation above the ground is consequently extremely dangerous.*

However, the fall can easily be checked by increasing

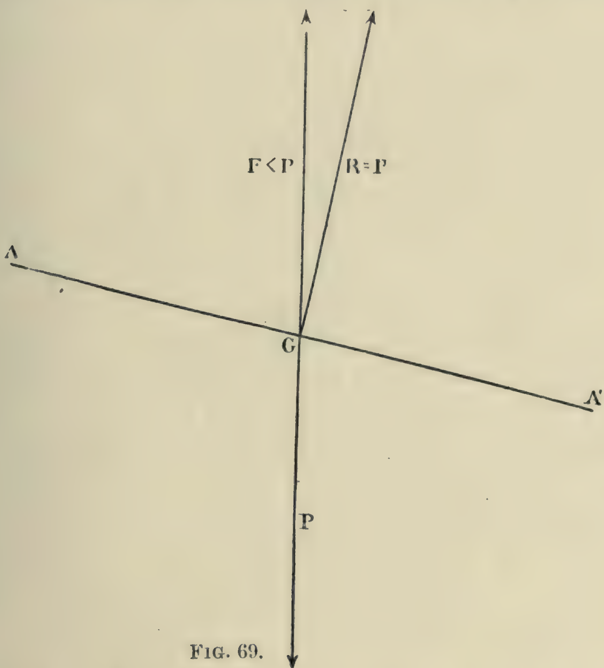

Fig. 69.

the angle of incidence and consequently the lift, while making a turn, by the use of the elevator.

This increase in the angle of incidence can be made automatic, either in the way mentioned at the end of § 65 if the turn is made with the rudder, or, if the machine is tilted for the turn, by placing the tail in such a position relatively to the axis of rotation, that the loss of longitudinal equilibrium due to the tilt (see end of § 60) causes an increased angle of incidence.

PART III

EFFECT OF THE WIND ON THE AEROPLANE

CHAPTER VI

THE WIND—REGULAR AND IRREGULAR WINDS—THEIR ACTION ON THE AEROPLANE

I.—The Wind

70. General considerations.

In all that has been written hitherto, both in Part I. and Part II., *the air has been assumed to be quite still.* But in practice this hypothesis can be said to be never realisable, and usually the aeroplane is subjected to the action of the wind, which at any given moment can be divided into the two entities of speed and direction.

A *regular* wind is one of which the speed and direction are constant. Any other wind is *irregular.*

It can be said that a regular wind does not exist, as its speed and direction are nearly always changing.

However, there is usually a mean speed and direction, from which the wind, except in a storm, does not alter very much, that can be made the basis of discussion. Such an irregular wind we will now consider.

Near the ground the contour of the earth sets up eddies which makes it impossible to lay down any law for the motion of the wind. But the higher one gets in the air, the disturbing elements which affect the mean speed and direction of the wind assume a certain regularity and

rhythm, and increasingly obey the great natural law of undulatory motion. Aerial waves are produced like waves in the sea, and at any fixed point a local increase of speed follows a decrease, which is succeeded in turn by a fresh increase—these phenomena following one another at relatively regular intervals.

The waves, or atmospheric pulsations, exist even when the air is comparatively calm; and when the wind rises they become squalls.

The direction of the wind must be considered not only in its horizontal motion but also in its inclinations to the horizontal. Roughly speaking, the wind tends to a horizontal direction the higher it is above the earth. One can, however, find ascending and descending currents.

Near the earth the wind generally follows the contour of the ground; vertical obstacles such as cliffs, crags, and woods for example, cause upward currents and eddies.

The idea mentioned above of considering an irregular wind as composed of a regular wind, the mean speed and direction of which is subject to disturbances, allows the study of effects of wind on the aeroplane to be divided into two parts.

First of all, we will take the action of a regular wind on the general motion of the machine, and afterwards the action of the various disturbances of the wind which can affect the aeroplane's equilibrium and stability.

II.—Action of a Regular Wind on the Aeroplane

70. Aerial vessels in a regular wind—The accessible circle and angle.

All machines capable of moving through the air—an aeroplane or a dirigible balloon, for example—behave in a regular wind in exactly the same way as in a dead calm. This point escapes most people, and it is important to examine it.

Aeronauts in a free balloon do not feel a breath of air, even if they are borne along by a very high wind. If a dirigible balloon is used, then, when the motor is started, they only experience and feel the wind which they thus create.

The dirigible balloon or aeroplane becomes part of the air which carries it along, and it is as if the air were motionless, in the same way as a fly flies in a carriage

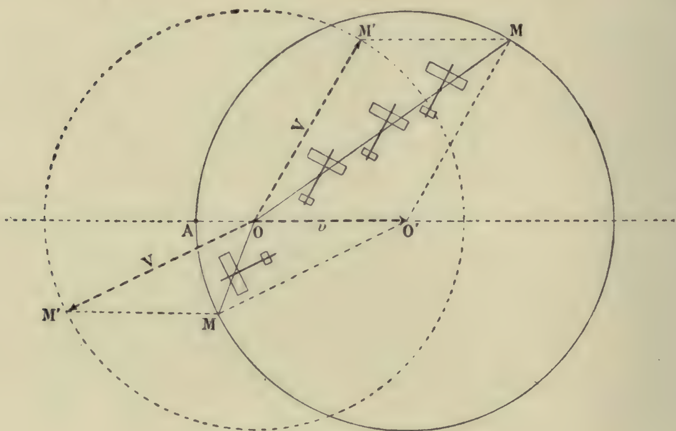

FIG. 70.

on the railway without perceiving or feeling the effect of the speed of the train.

By the aid of an idea due to Colonel Renard, which he termed "the accessible circle and angle," the effect of a regular wind on the movement of any aerial vessel can be determined.

If such a vessel starting from the point O (Fig. 70) in quite calm air has an independent speed V, it will arrive in the lapse of one second at some point, according to the direction taken, on the circumference drawn with O as centre and V as radius. (The circumference is dotted in the figure.)

If, on the contrary, the vessel encounters a regular wind of the speed v, the circumference, on which it will find itself after one second, will be that of the radius V drawn from the centre O′ so that OO′ will be equal in length and direction to the speed v of the wind. (The circumference is drawn as a line in the figure.)

Again, if the pilot wished to go from O to M, he will head in a direction parallel to OM′, and to any observer who, like him, is being carried by the wind (for instance, an observer in a free balloon), the pilot will appear to go in the direction OM′. The line OM will be the *real path* of the vessel and OM′ its *apparent trajectory*.

In fact an aerial vessel can only reach in a second points situated inside the circle with the centre O′, which for this reason is known as the *accessible circle*.

When the circumference of this circle includes the point O′, that is, when the independent speed V of the vessel is superior to the speed v of the wind (as in Fig. 70), the vessel can move in all directions around O, and particularly can go against the wind, that is, from O to A. And though its actual speed is only $V-v$ it can operate effectively and is actually dirigible.

If, on the other hand, the speed of the wind is greater than the independent speed of the machine, the accessible circle does not include the point O (Fig. 71), and the vessel cannot, when starting from this point, proceed in any direction except those included in the angle BOB′, formed by tangents drawn from the said point O to the accessible circle.

To go, for instance, from O to M, the machine must head in a direction parallel to OM′. If turned head to wind it will (in one second) only succeed in reaching the point A; in other words, it will be driven backwards. Such a machine is not then dirigible; the most that can be done is to alter the angle of its flight-path with respect to the direction of the wind.

The angle BOB′ limits, according to the direction of the wind, the paths to be taken by the aeroplane; for this reason it is called the *accessible angle*.

The *mean* independent speed of aeroplanes at the present moment is from 75 to 80 km. per hour, and it will without doubt increase in the future.

Hitherto aerial vessels have been unable to go out except in comparatively light winds, but the aeroplane can

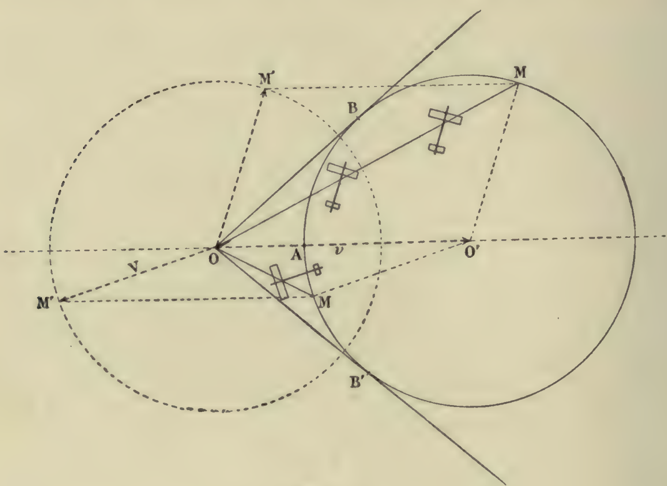

FIG. 71.

be included in the first category of machines mentioned above; it is dirigible, and will become more so.

72. Effect of a regular horizontal wind on a glide.

A regular horizontal wind acting on a glider in motion has the effect of altering the angle of glide, the result differing as the wind is in front or behind the glider.

Let OX (Fig. 72) be the flight-path, in still air, of a glider starting from O. At the end of one second the glider will arrive at A, and its horizontal speed will be measured along the line *a*A.

If, instead of being in still air, the glider is carried forward by a following wind of the speed AA_1, its actual path will be OA_1, or more nearly horizontal than OA.

This explains how certain birds gliding along with a high wind do not appear to drop appreciably. They must, however, come down, provided the wind is regular and absolutely horizontal, for its speed would have to be infinite to prevent this.

On the other hand, if the glider comes down head to wind in a current of the speed AA_2, its actual trajectory will be OA_2, more abrupt than OA. This trajectory would be vertical if the speed of the wind equals the mean horizontal speed of the glider, and an observer on the ground would see the glider slowly descending vertically [1] at a speed that

FIG. 72.

would be less as the angle of incidence approached the economic angle (see § 40) at which the air can be held for the longest possible time.

It should be remarked that *the aviator can only become aware of the variations of his actual flight-path by watching the ground.*

The machine always follows its path in respect to the wind, and lies wholly on it; in consequence, if its angle of incidence has not been altered by the elevator, the glider, when it follows the different actual paths OA_1, OA_2, Oa,

[1] This species of descent, which is at present made only, and then not often, by birds (for aviators do not go out when the wind speed equals the mean speed of their machines), must not be confused with the parachute descent (§ 60, footnote 2). Both, however, would look just alike to an observer on the ground, the actual difference being that the first is made against the wind and the second in calm air.

always remains parallel to the position which it assumed on the path OA in calm air.

The flight-path is, so to speak, compelled by the wind, the glider continuing in the same path as if no wind existed.

The foregoing considerations explain why it is better to land facing the wind (see § 46), which acts as a brake on the machine.

73. Effects of ascending currents on the glider.

If a glider, starting from the point O (Fig. 73), is capable in calm air of reaching the point A in the space of

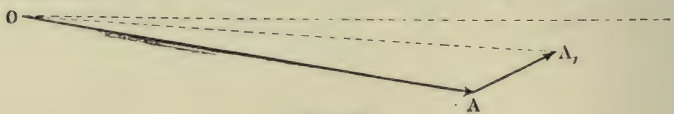

FIG. 73.

one second, if it were subjected to the action of a regular ascending wind of the speed AA_1, it would be, at the end of the same time, at the point A_1, and its actual flight-path would be OA_1.

It follows that, *given a certain speed and direction of the ascending current, the machine would glide without falling.*[1] In calm air, of course, this could not occur (§ 41), nor in a regular horizontal current, as gravity must always be served.

[1] This result can also be brought about by a head wind (Fig. 73A).

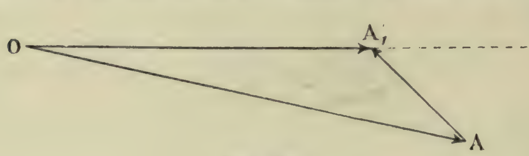

FIG. 73A.

Experiment with model gliders is quite interesting, for they can be got to advance against the wind, which must, however, be ascending as well.

Ascending currents have been quoted to explain the soaring flight of certain birds, but we shall see later on in § 75 that this explanation is inadmissible except in special cases.

III.—The Action of Irregular Wind on the Aeroplane

74. General considerations.

The disturbances to which the aeroplane can be subject consist generally (see § 70) in changes in the speed and direction of the wind. They can be represented, therefore, by arrows or vectors of given size and direction.

Whether an aeroplane is flying in calm air or is being carried along by a regular wind, the action of a disturbing force is divisible, like all mechanical actions as applied to solid bodies, into two parts: one being a force tending to alter the motion of its centre of gravity, the other tending to turn it around an axis passing through this point.

The first of these primary forces may be compared to the effect of regular wind. It changes the direction of the actual flight-path, but not that of the apparent one. In other words, it seems as if the mass of air surrounding the aeroplane moved with it. Furthermore, an aeroplane flying horizontally and struck by an up-current, will be bodily lifted up without the pilot being conscious of it, but only if the wind performs the function of the force under consideration and nothing more.

In this case unless the pilot is watching the ground he will not feel his sudden elevation, and will imagine he is still pursuing—as his machine remains horizontal—his original path.

The second force, that of the moment, alters the equilibrium, which in turn alters the direction of the aeroplane. This time the apparent flight-path is involved, and the aviator can feel and correct the changes.

To form some idea of these effects, one must suppose, as previously, that the aeroplane is suspended by its centre of gravity.

If the vector OA (Fig. 74) represents in extent and direction the speed V of the aeroplane, the speed of the

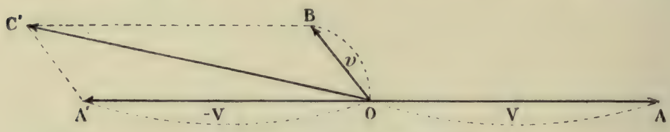

FIG. 74.—Plan or elevation.

wind striking the machine is represented by the vector OA′, equal and opposite to OA. If, on the other hand, the vector OB is taken as the speed v of the disturbance, a new speed OC′ of the opposing wind compounded from the vectors OA′ and OB is obtained.

The diagram of Fig. 74 can be applied to currents both

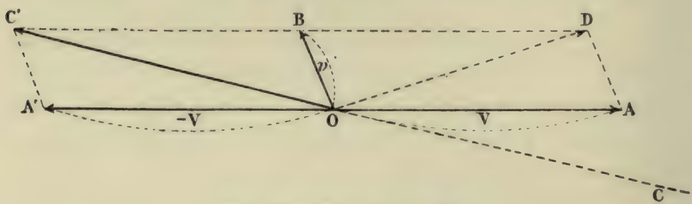

FIG. 75.—Plan or elevation.

upward and horizontal (though not necessarily horizontal to the direction of motion of the machine).

It will be clear from the fact that the machine rolls and pitches along the line taken by the opposing wind that the second force mentioned above has the effect of deflecting the apparent flight-path.

Both the first and second forces may combine in a single gust of wind. Taking then the arrangement of Fig. 74 and completing it as in Fig. 75, in which OD equals the wind speed taken by the aeroplane under the

effect of the first force, it appears that the flight-path is pushed two ways at once to OD and OC, OC being directly opposed to the new direction of the wind OC′. The actual path resulting from the total effect is consequently but little different from that which the machine was originally following. The aviator will feel that his trajectory has altered, but as, at the same time, he will be carried away without his knowing it in a direction opposite to that of the apparent change, his path will only be slightly deflected.

The mass of an aeroplane considerably affects its behaviour with regard to wind gusts. If it is light it will almost immediately conform to the new speed; if, on the contrary, it is heavy, the time taken to acquire a different speed will be appreciable; and during this time the air exerts pressures on it similar to those which it would experience if, instead of being entirely free, it found itself partially impeded in motion. Such pressures are utilised to economise power by heavy birds, but at present to an aviator they are rather detrimental, though perhaps some day they may be of use to him.

The effects on the aeroplane of atmospherical disturbances or gusts of wind will be treated successively in the following special instances:—

1. The action in horizontal flight of a gust of wind directed along the flight-path.

2. The action in oblique flight of a horizontal gust of wind.

3. The action in horizontal flight of a horizontal gust of wind from the side.

4. The action in horizontal flight of an oblique gust of wind.

75. The action in horizontal flight of a gust of wind directed along the flight-path.

It is necessary here to distinguish the effects of the

two elementary forces defined in § 74: the change of the actual trajectory of the centre of gravity and the change of longitudinal equilibrium.

The effect of the first force is as follows:—

If a gust of wind strikes the aeroplane head-on, the lift is increased and the machine rises. When the disturbing force strikes it from behind the aeroplane falls, and this effect can be considerable if the speed of the gust is equal to the mean speed of the machine. If the speed of the wind is greater, the lift momentarily vanishes and becomes negative, the plane finding itself struck from the rear and from above.

We may, therefore, say that:

It is advantageous to increase the mean speed of an aeroplane flying in disturbed air.

With regard to the effect of the second force, if the wind attacks the main plane and the tail simultaneously, the longitudinal equilibrium will not be altered, for it is independent of the value of the speed (§ 62), but a gust of wind which is a kind of wave proceeds like all undulating motions at a definite speed; it therefore first strikes the front plane and upsets the longitudinal equilibrium, and then, passing along to the rear plane, creates either another loss of equilibrium in the opposite sense or an accentuation of the first. The result is a pitching motion, which can become dangerous if the plane arrives at the condition of being struck from above.

For this reason, when the main plane is placed in front, it is as well to bring as close together as possible the centre of pressure corresponding to the normal angle of incidence and the (projected) centre of gravity. This negatives to a large extent the pitching effect. In the case where the two points exactly coincide, the non-lifting tail (see §§ 52 and 54) remains parallel to the path of the wind, and is consequently barely affected by the gust.

After what has just been said, it is evident that *the*

nearer the main plane and the tail are together the more quickly the gust passes by the machine, and consequently the loss of equilibrium is smaller. But as we have seen that in calm air the stabilising effect of the tail is proportional to its size and distance from the main plane, it would appear of advantage, when bringing it closer to the main plane in order to decrease the effects of wind gusts, at the same time suitably to enlarge its surface. The result would be to produce an aeroplane shorter in its longitudinal dimension than those now existing. Nature herself furnishes an example of this: good flyers among the birds are much shorter proportionately than modern aeroplanes. We may therefore say that:

Long bodies, which in calm air contribute to excellent longitudinal stability, are much less advantageous, and can even become detrimental, in disturbed air.

It was this fact that led the Wright Brothers to neglect automatic longitudinal stability almost completely, and to construct the tail of their machine as a single movable organ, which was also the elevator. As this device was placed in front, the aviator could as it were see the gust coming, and prepare the main plane to meet it by a suitable alteration of the angle of incidence. On the other hand, longitudinal equilibrium must be constantly preserved, even in calm air, by the action of the pilot, to whom a false movement or even momentary inattention might be fatal. The machine, however, is remarkably "tender," and responds to the smallest prompting of the pilot.

By way of comparison, it may be said that the longitudinal stability of a tailed aeroplane is like that of a bicycle, while the Wright aeroplane similarly corresponds to a monocycle.

After all, the best results are brought about by furnishing the aeroplane with a reasonable tail and using a

forward elevator to damp out fore and aft oscillations. This arrangement is used on a large number of machines.[1]

A method of preserving, automatically, longitudinal stability against wind gusts has naturally been much sought after, and the experiments made hitherto are divisible into two classes:

1. Those which are intended to correct the oscillation after it has started.
2. Those to prevent any oscillation whatsoever (which appear, *a priori*, to be preferable).

The first class is based on the automatic control of the elevator either by a pendulum or by a gyroscope, the idea being to produce a fixed lever point which is unaffected by the oscillation of the machine. But at the present moment no results of any value have been obtained.

From the second class better things may be expected.

In this case the idea is to use feelers (*palpeurs*) or a sort of antennæ something like weathercocks, which would encounter the gust before it reached the main plane and prepare it to meet it by altering the angle of incidence through the medium of a control connecting them with the elevator.

Whatever may ultimately be the means employed, it seems likely that aviators will not have to busy themselves with maintaining equilibrium, and that the aeroplane of the future will fly by itself so stably that one will believe it, in M. Soreau's phrase, "to be guided by invisible rails."

Wind gusts have more effect on an aeroplane as its mean speed increases.

If the speed V of an aeroplane is subject to an acceleration v, the pressure of the air upon the planes, since it varies as the square of the speed, increases proportionately

[1] In the modern type of Wright biplane the combined elevator and tail, which have been simplified into a single plane, of which the trailing edge is capable of being warped up or down, is placed behind the main planes in rear of the vertical rudder.

as $(V+v)^2-V^2$, or if v^2 is disregarded as negligible, as $2Vv$. The effect of the acceleration v on the aeroplane is, of course, greater as the mean speed V is greater. This explains why, when the wind seems slight, aviators constantly encounter gusts.[1]

It must not, however, be imagined that because the disturbing effect of a gust of wind increases with the speed of the aeroplane encountering it, that longitudinal stability decreases as the machine's speed increases. On the contrary, the effect is proportional to the speed, and the righting moment which it sets up is proportional to the square of the speed.

The stability of an aeroplane, therefore, increases with its speed.

We will conclude this section with a brief discussion of the fact that certain large birds, by making use of the intermittent flow of the air, can support themselves in the air without apparent motion. The most reasonable explanation of this is that of M. Soreau, who terms this particular form of flight, which some species practise above wide expanses such as seas and deserts, "soaring flight."

When the bird experiences a gust, its inertia momentarily resists it, and it immediately places its wings at such an angle, that the effect of the gust is to raise it. As soon as the gust passes and is succeeded by a comparative lull, the bird again alters the angle of its wings and glides down at the most economical angle until it encounters a fresh gust.

It is evident that birds use ascending currents when they come across them, but one can scarcely admit that the existence of the currents, which must supposedly be

[1] These gusts, by the way, are more frequent when the machine is flying into the wind than with the wind, for the simple reason that the gusts have a certain speed, and the aeroplane therefore meets more of them, in the same space of time, when flying into the wind than when travelling with it.

permanent, sufficiently explains the phenomena of "soaring flight," which can be prolonged for several hours.

In our latitudes, moreover, one can readily observe birds of prey rising in a wind without any apparent movement of their wings by describing circles, and they all, when flying together, seem to rise and circle at the same rate of speed.

Perhaps the period of their circles coincides with the rhythm of the gusts, and they turn when the gust is succeeded by a lull, so as to convert the effect of the lull into that of a gust, thereby utilising to the full the intermittent waves of the air.

76. The action, in oblique flight, of a horizontal gust.

When, instead of flying horizontally the aeroplane flies obliquely, the two primary actions (§ 74) of a gust horizontal to the flight-path (that is to say, situated in the same vertical plane) are the same as the foregoing. At the same time, the variation of the pressure on the plane depends not so much on the strength of the wind as on the inclination of the flight-path.

The general idea in § 74 can, in fact, be applied in the case of ascending flight (Fig. 76) as well as in descending

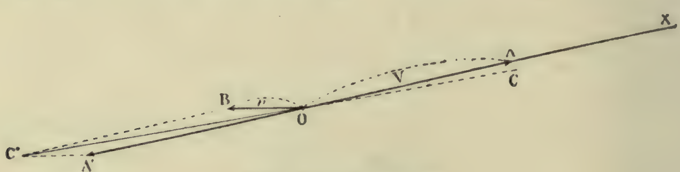

FIG. 76.—Elevation.

flight (Fig. 77). If OX is the direction of the flight-path, the effect of a gust of wind from the front, OB $= v$, will be to direct the relative wind striking the machine along the line OC′. This will result in an alteration, COA, of the

angle of incidence, increasing it if the machine is rising, and diminishing it if the machine is descending.

This alteration will be greater for the same increase of the speed v as the inclination of the plane is more pronounced and as the mean speed of the machine is less.

The pressures, therefore, on the planes will change not only from variation of the speed, but from the alteration

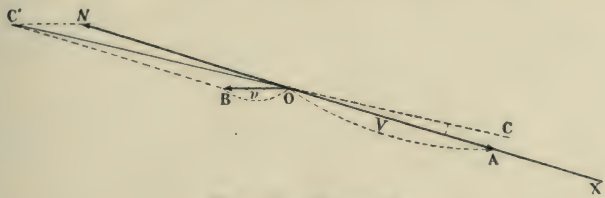

FIG. 77.—Elevation.

of the angle of incidence. This latter alteration will be inverse in the case of a following gust.

To sum up:

An aeroplane is more affected by horizontal gusts when flying obliquely than when horizontally, and the more so as its mean speed is less.

77. The action, in horizontal flight, of a horizontal lateral gust.

When the aeroplane, flying horizontally, is struck from the side by a gust of wind also horizontal, the two primary actions of this gust have, generally speaking, the same effects as in § 74, and the diagram of Fig. 75 (reproduced as Fig. 78) is applicable, taking into consideration the altered conditions.

If OA is the mean speed V of the machine and OB that of the wind gust v, the direction of the wind experienced by the aeroplane will be OC′, and, supposing it to possess directional stability, it will turn head to wind and take up the apparent trajectory OC. The aviator will

be quite aware of this movement, and can, by the aid of a compass, measure the amount of deviation.

But, at the same time, the effect of the first primary action of the wind will be to give the aeroplane the actual

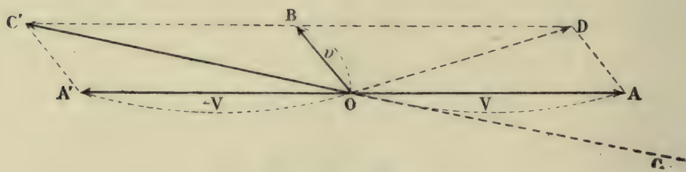

FIG. 78.—Plan.

trajectory OD, of which the aviator will know nothing.[1] As a matter of fact, the actual path of the aeroplane as observed from the ground will be very little changed.

The second primary action, the moment of the wind, will cause the machine to swing round horizontally.

Taking the speed and direction of an aeroplane to be constant, one can easily find out in what direction the

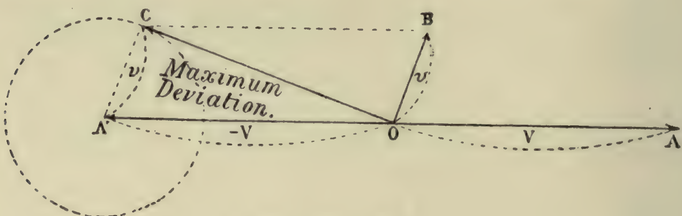

FIG. 79.—Plan.

gust should blow in order to create a maximum horizontal swing.

In applying Fig. 74 it is clear that the direction will be perpendicular to the tangent drawn from the point O (Fig. 79), to the circumference described with A′ as the centre and the speed, v, of the gust as radius. If the circumference includes the point O (Fig. 80), that is, if the

[1] Unless he is watching the ground.

speed of the gust is more than the speed of the machine, the deviation will amount to 180°. In other words, an aeroplane struck from behind by a gust, the speed of which is greater than that of the machine, will turn completely round like a weathercock greatly to the danger of the pilot, and it will do so the more quickly in proportion as its directional stability is greater.

As mentioned in § 75, in this case the planes can be struck from above also. For both these reasons it is de-

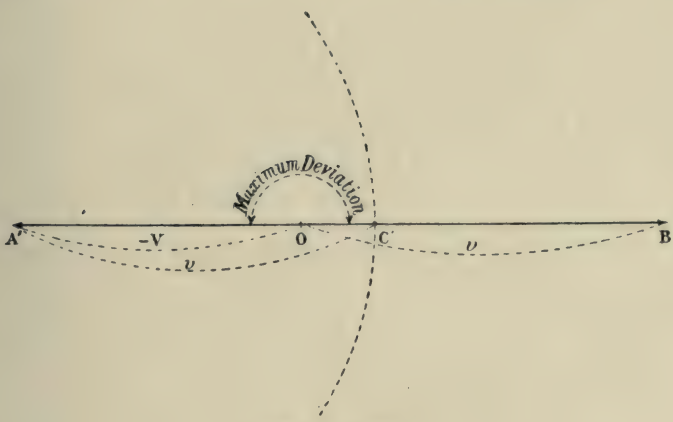

FIG. 80.—Plan.

sirable to increase the mean speed of the aeroplane, and to reduce directional stability to the minimum consistent with efficiency. Too much importance should not, however, be attached to this extreme case, which would be very rare in practice. But *aviators should not risk being struck by gusts of a speed greater than that of their machine.* As the speed of the aeroplane is increased, the possibility of this will gradually disappear.

It has been seen that by the application of Fig. 74, the effect of the second primary action of a wind gust on the apparent path of the aeroplane could be determined, that

is, the effect of the moment of the reaction due to the gust in respect to the principal axis of inertia which is practically vertical.

But this reaction can also affect the principal longitudinal axis of inertia, or what has been called in § 58 the axis of rotation. In such case the lateral equilibrium will be upset, the machine will tilt and oscillate in a way which may become dangerous.

In the first place the initial tilt of the machine will cause a tendency to turn. If the pressure this tendency exercises on the keelplane is above the axis of rotation, the turn will be made by the machine in the opposite direction to that in which its directional stability would turn it. Generally the effect of the turn is the stronger; the aeroplane drifts before the wind, and, being side-on, its loss of equilibrium due to the action of the gust of wind is accentuated.

On the other hand, if the pressure is below the axis of rotation, the tendency to turn keeps the aeroplane in the wind and damps out the disturbing influence. This can be seen in sea-birds, which place their wings in the form of an inverted dihedral (see also Fig. 64, § 60).

From this point of view it would appear advantageous to adopt a similar form for planes, or to lower the centre of the keelplane. But this can only be done (§ 60) when the axis of rotation occupies a certain position in respect to the plane, and it is usually considered to upset the lateral stability.[1]

Usually, a lateral gust is not absolutely perpendicular to the plane of symmetry, and consequently it introduces a component parallel to the direction of the flight-path (see § 75).

[1] However, in future, as aeroplanes improve, the wings may possibly form a slight inverted dihedral. The centre of gravity should then be placed quite low, so that the axis of rotation forms with the plane an angle greater than the angle of incidence.

One can, therefore, say that a lateral gust of wind, as a rule, disturbs simultaneously both the lateral and the longitudinal stability, and gives rise to pitching[1] and rolling oscillations.

We can say, as we did in the case of longitudinal equilibrium, that:

Arrangements which in still air insure lateral stability, such as planes placed at a dihedral angle or a high keelplane, are most disadvantageous in disturbed air.

For this reason many constructors have not hesitated to sacrifice almost entirely any guarantee of automatic lateral stability, by placing the centre of the keelplane at a very small distance above the axis of rotation and by abandoning the dihedral-angled plane. Of the Wright machine, particularly, it can be said that it has no transverse (as it has no longitudinal) stability. However, the aviator must be provided with some means for dynamic stability. These have been described in § 63 (warping, balancers, &c.).

The same methods used or suggested for longitudinal stability, such as the pendulum,[2] gyroscope, "feelers," &c., may be employed in transverse stability, and though none of these means have yet yielded any positive results, the problem will doubtless be solved in the near future.

78. The action, in horizontal flight, of an oblique gust of wind.

When an aeroplane, flying horizontally, is struck by an oblique gust of wind, as happens when it encounters an ascending or descending current or an eddy, the same pro-

[1] Independently of those which (see § 60) result from rolling oscillations.

[2] The procedure briefly discussed in the second footnote to § 63 is included in the category of those which have the action of the pendulum for their principle, since the weight of the machine causes it to be displaced, whence the righting moment is created.

cedure applies as used heretofore, and Fig. 75 (reproduced in Fig. 81) is again applicable.

If OA is the mean speed V of the aeroplane, and OB the speed v of the gust, the new direction of the wind will be OC′. The machine will therefore tilt, and tend to take an apparent trajectory OC. Of this the pilot will be conscious.

But at the same time the effect of the first primary action will result in its following the actual path OD, of which the aviator will not be aware. The path of the machine, viewed from below, will not therefore be much altered.

But this is not all, for the gust strikes the forward

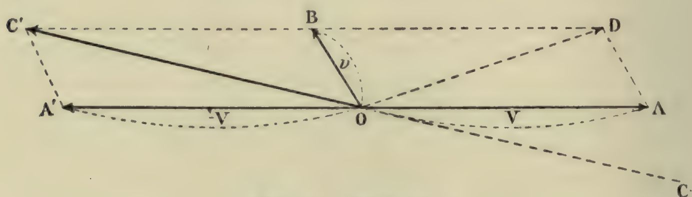

FIG. 81.—Elevation.

plane first and upsets the longitudinal equilibrium; the two primary actions of the disturbance both[1] tend to make the machine rise, and then as the forward plane gets clear the gust meets the other, and another loss of equilibrium in an inverse sense occurs. In fact, such a gust creates both a considerable pitching motion and a vertical undulation of the flight-path.

It is therefore important, with a view to diminishing these detrimental motions so to construct and handle the aeroplane, that the pressure on the main plane should, under normal conditions of flight, pass through the centre of gravity.

[1] This will not be so if the aeroplane has a tail (Fig. 59) which is struck by the wind on its upper surface.

79. Conclusions.

The motion of the aeroplane in an atmospherical disturbance is in reality a combination of those taken in the foregoing simple cases, for it is very seldom that a gust can be exactly classed in any one of the categories mentioned, and usually it is composed of two or more of them.

But it has been made clear that:

1. The aeroplane is less affected by gusts in proportion as its speed increases.

(The machine of the future should therefore fly at high speeds.)

2. Arrangements that insure automatic stability in calm air are not suitable or useful in disturbed air.

From this comes the tendency of several constructors, such as the Wrights, to insure equilibrium by indifferent equilibrium and dynamic control. Machines constructed on this principle hold the wind better than others, and are very responsive and manageable; moreover, the suppression of big tails and large keels increases their fineness (§ 14). But to drive them, whether in calm or disturbed air, makes a constant demand on the powers of the pilot, and the slightest inattention might cause an accident.

The foregoing considerations show the considerable interest that attaches to the solution of the automatic stability of an aeroplane in disturbed air.

Fear of the effects of the wind will not always oblige the sacrifice of the guarantees for security which the present-day machines possess as far as calm air is concerned, and aviators will yet go out, when the wind is not too strong, without experiencing a continual and dangerous struggle against it. Then only will the aeroplane become a really practical means of transport.

This consummation is nearer than most people think. Who knows, too, if man will not one day go out and brave, dominate, and subdue a tempest!

PART IV

PROPULSION

CHAPTER VII

THE SCREW-PROPELLER

80. Definition of the Screw—Pitch—Thrust—Slip. To utilise the power of the motor for producing the force necessary to sustain the aeroplane, the screw-propeller [1] is employed.

One can briefly consider it as acting like a screw. It is composed of a number of arms or *blades,* which are usually sectors of a screw surface.

If it acted exactly like a screw entering a solid body it would advance in one turn a distance equal to the pitch of the screw, and therefore this distance is known as the *pitch of the propeller.*

But as the air is essentially volatile, the same result does not obtain in it as in a solid body. The blades, to produce a *thrust,* should attack the air in the same way as the planes of a flying-machine. It follows that the propeller only advances a distance appreciably less than its pitch for each complete turn. In this way slip is produced, which is as indispensable for the creation of the thrust as the angle of incidence is for the lift of an aeroplane.

[1] It is not impossible that in the future other means of utilising the motive power will be employed, such as the vertical beating of the carrying planes, which is not so impracticable as some affirm. The vertical oscillation of a heavy mass would effect this, and Captain Etévé has given the name of ornithoplanes to machines which can be constructed on this principle.

81. The angle of incidence of a portion of the propeller-blade varies with the forward speed of the propeller—The actual angle of incidence—The apparent angle of incidence.

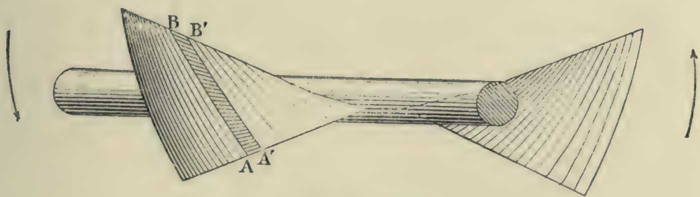

FIG. 82.—Perspective.

In all that follows, the term "portion of the propeller-blade" will be applied to a section of the blade ABA′B′ (Fig. 82) narrow enough to be considered as a plane.

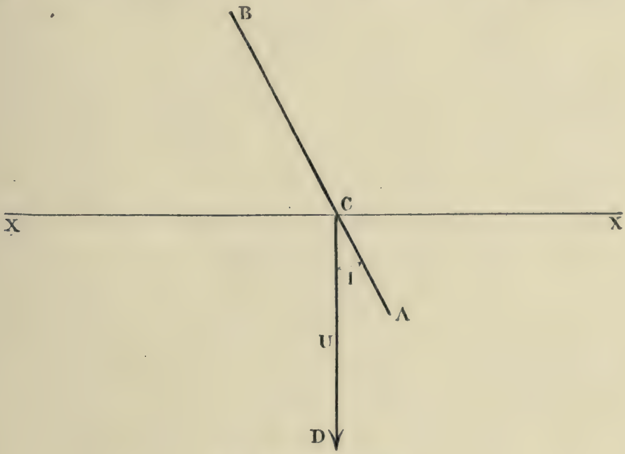

FIG. 83.—Profile.

The first principle to be learned in the study of propellers is as follows:—

The angle of incidence of a portion of the blade, that is, the angle at which the portion bites the air, is smaller than it appears to be when the propeller

moves forward when revolving, and it becomes less as the propeller's forward speed increases.

Let XX (Fig. 83) be the horizontal axis of a screw-propeller, and AB the section of a portion of the blade viewed from the tip, this portion being at a distance $\frac{d}{2}$ from the axis.

If the propeller revolves without advancing, that is, if it is stationary, the point C describes a circumference at

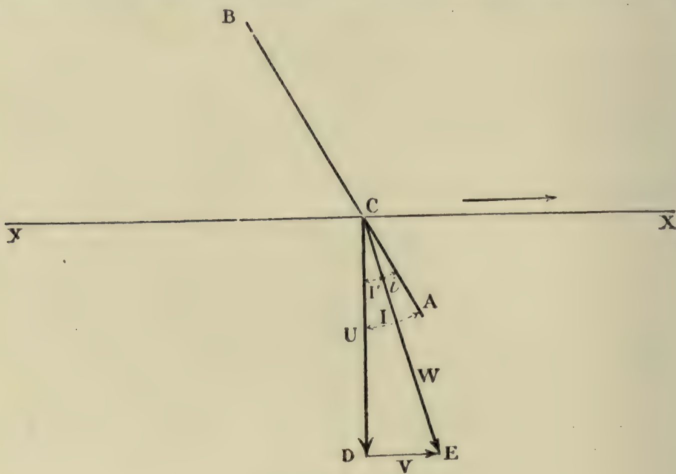

FIG. 84.—Profile.

a linear velocity CD, or U, of the value πnd, n being the angular speed in revolutions per second. AB therefore seems[1] to attack the air at the angle ACD.

If the propeller, revolving at the number n of revolutions per second, advances with the speed V (Fig. 84), the velocity of the point C is no longer CD but CE, or W, the resultant of the added speeds U and V. The angle of incidence of AB is therefore ACE, smaller than the angle ACD, and it grows less as the forward speed V of the propeller increases.

[1] See footnote, next page.

This idea is extremely important, and must be thoroughly grasped before proceeding further.

The angle ACE will be called the *actual angle of incidence of the portion of the blade* and represented by the symbol i.

The angle ACD will be called the *apparent angle of incidence*, because it is that at which the portion appears [1] to meet the air, and will be represented by the letter I. This angle is also the complement of the angle ACX which the chord of the portion makes with the direction of the propeller axis. For this reason the angle I will be frequently mentioned as the *inclination* of the portion of the blade.

If the propeller entered a solid body instead of air, the point C would follow the path CA (Fig. 84).

Since during one revolution C travels in the direction CD, from the fact of rotating, a distance equal to that, πd, of the circumference of the circle it describes, and, at the same time, in the direction CX, from the fact of forward motion, a distance equal to the pitch H of the propeller, the ratio $\frac{H}{\pi d}$ of these two distances gives the value of the apparent angle of incidence or pitch, ACD, of the portion of the blade. This value depends at the same time on those of the propeller pitch H and the distance d of the said portion from the axis.

Reference to Fig. 85—a view in perspective—makes it clear that *the nearer the portion of the blade is to the axis the greater the inclination.*

The angle ECD (Fig. 84), made by the speed W and the speed U, which will be called I′, can be measured by the expression $\frac{V}{U}$ or $\frac{V}{\pi n d}$. Its value, therefore, depends on

[1] Similarly when stationary, the real angle of incidence of the portion of the propeller-blade is smaller than its apparent angle, because the propeller, acting like a ventilator, works permanently in the air-current which it creates.

that of the ratio $\frac{V}{n}$ of the forward motion of the propeller to its velocity of rotation, and also on that of the distance d of the said portion from the axis.

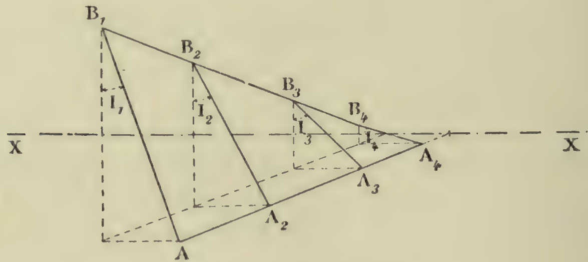

FIG. 85.—Perspective.

The actual angle of incidence i of the portion of the blade is equal to the difference between the angles ACD and ECD:

$$i = I - I'.$$

Taking a known propeller revolving and advancing

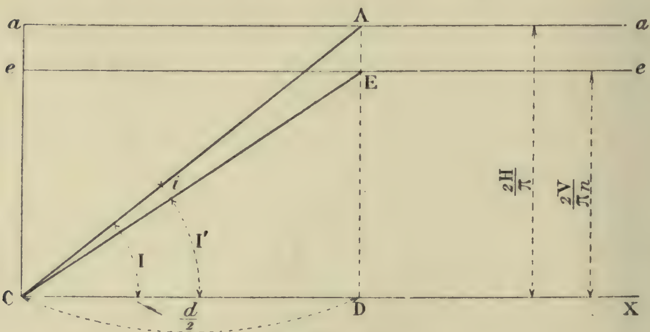

FIG. 86.

at known speeds, thus supposing H, V, and n (or rather H and $\frac{V}{n}$) constant, one can represent graphically, as in Fig. 86, the variations of the two angles I and I′ proper

to each portion of the blade, and also that of the actual angle of incidence i (the difference between I and I′) with the difference of each portion from the propeller axis.

If above the horizontal line CX two lines *ee* and *aa* parallel to it are drawn at the distance respectively of $\frac{2V}{\pi n}$ and $\frac{2H}{\pi}$, then to each point D, situated on the horizontal CX at a distance $\frac{d}{2}$ from C, there correspond two angles ECD and ACD, the first of which is equal to $\frac{V}{\pi n d}$ and the second to $\frac{H}{\pi d}$. Consequently these angles are equal to those designated by the same letters in Fig. 84—in other words, to the angles I′ and I.

The angle ACE therefore represents the actual angle of incidence i of the portion of the blade at a distance $\frac{d}{2}$ from the propeller axis.

The geometrical study of this shows that the actual angle of incidence is variable for every point on the blade, and reaches a maximum value at a definite distance from the axis.

82. The thrust and resistance to rotation of a portion of the propeller-blade.

From the foregoing considerations it will be apparent that the problem of the propeller is very similar to that of the planes of the aeroplane.

Each portion of the blade AB (Fig. 87) at the angle i creates by the air pressure a reaction R, proportional both to the said angle i and to the square of the speed W of the point C. This reaction R can be divided into two parts, j the thrust, operating parallel to the propeller axis, and q *the resistance to rotation*, operating perpendicularly to this axis.

N

The useful work of the propeller is the thrust—the total thrust J is the sum of all the small thrusts created by the various portions of the blade.[1]

The resistance to rotation q includes, in the same way as the resistance of a plane, an active part, inseparable from the creation of the thrust, and a part entirely passive due to the thickness of the blade and the skin friction on its surfaces.

This latter part should be diminished as much as

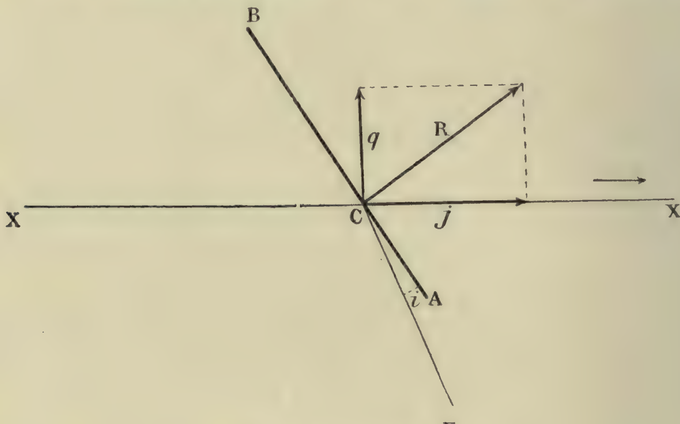

FIG. 87.—Profile.

possible by putting a fine finish on the blades and eliminating projecting ribs.

Moreover, wide blades are quite useless, in spite of some inventors who would have us use Archimedean screws as propellers, for if the blade is too wide the skin friction on the surfaces increases much more rapidly than the thrust. A propeller-blade, like the planes of an aeroplane, should have a large aspect ratio. Reference to Fig. 38 (§ 48) shows that the trailing edge of a plane gives very little lift and usually creates a suction.

[1] It is quite possible, however, that every portion does not work as it would if it were a separate entity.

83. Efficiency of a portion of the blade—The optimum angle—The maximum maximorum efficiency.

We know (§ 18) that the useful power required for sustentation is expressed by the product Vt of the speed V of the aeroplane and the tractive effort t, which is the thrust J of the propeller. The necessary useful power has, therefore, a value VJ, the product of its forward speed and the thrust.

Every part of the blade participates in the production of this useful power, and its quota is measured by the product Vj, wherein j represents the thrust of the said portion.

On the other hand, in order to rotate the portion of the blade at the number of revolutions per second n, it is necessary to overcome the resistance q which the air opposes to its rotation by communicating to the point of application C of q a circumferential velocity U, of which the value is πnd. Thus a motive power is applied to the portion of the blade under consideration equal to πndq.

The ratio $r=\frac{Vj}{\pi ndq}$ of the useful power furnished by the portion of the blade and the motive power employed to produce it, represents the efficiency of this portion. The relation can also be put in the following form:

$$r=\frac{j}{q}\mathrm{I}',$$

I′ representing the angle defined in § 81 whose value (equal to the difference $\mathrm{I}-i$ between the apparent angle of incidence and the actual angle of incidence proper to the portion of the blade under consideration) is measured by the expression $\frac{V}{\pi nd}$. The efficiency of a portion of the blade has the final expression:

$$r=\frac{j}{q}(\mathrm{I}-i).$$

Supposing the inclination I *of the portion of the blade to be given,* the value of the thrust j and the resistance to rotation q and also that of their relation $\frac{j}{q}$ depend only on the value i of the actual angle of incidence. Therefore the same applies to the value r of the efficiency of the portion of the blade.

Pursuing the subject further, we find that:

The efficiency of a portion of the blade, taking its inclination I to be always constant, reaches a maximum when the actual angle of incidence i has a certain value i_1.

When, taking another value I′ of the inclination, one wishes to find out in a similar way the value of the actual angle of incidence which gets the highest efficiency out of a certain portion of the blade, *one always finds it to be the same value* i_1.

The efficiency of every portion of the blade is at its maximum when the forward speed and the velocity of rotation of the propeller are such that the actual angle of incidence has the single value i_1.

The value i_1 varies in every propeller, and is called its *optimum angle.*

If the value i_1, of the actual angle of incidence, which gets the maximum efficiency out of the inclination I of a portion of the blade, is independent of the value of this inclination, the same does not apply to the value, r_1, of the said maximum efficiency. To each value of I there corresponds a value of maximum efficiency obtained at the angle of incidence i_1, but *it changes with the value of the inclination* I, which at about 45° (theoretically) gives the greatest maximum value of efficiency. Therefore:

There is a portion of the blade whose maximum efficiency is maximum maximorum.

This portion must conform simultaneously to the two following conditions:

1. It must actually meet the air at the optimum angle i_1.
2. It must possess an inclination in the neighbourhood of 45°.

Of these conditions, the first is by far the most important, and it would be useless to give the portion of the blade such an inclination if it did not actually meet the air at the optimum angle.

84. M. Drzewiecki's variable-pitch propellers.

We have seen in § 81 that when the pitch H of the propeller and the relation $\frac{V}{n}$ of its forward speed to its rotary velocity remain constant, the actual angle of incidence varies with the distance from the axis of the portion of the blade under consideration.

It is not therefore possible, if the propeller is a true screw, that is, *with a constant pitch,* for each portion of the blade to give out its maximum efficiency, for if the actual angle of incidence is the best for any one portion, it will be so for it alone, and all the others will have angles differing from the optimum angle by the distance they are away from the aforesaid portion of the blade.

M. Drzewiecki, with the idea of getting the maximum efficiency out of every part of the propeller, varies the pitch at each point so that the actual angle of incidence is everywhere the optimum angle.

Propellers of this kind are known as variable-pitch propellers, and are consequently no longer true screws.

Although a maximum efficiency may be obtained from each part of these screws, the value (see § 83) is not the same for every part. It is greater in proportion as the inclination of the part more nearly approaches the inclination of maximum maximorum efficiency or the neighbourhood of 45°.

85. Total efficiency of a propeller.

Each portion of the blade of a propeller gives out a thrust j, and the sum of all these forces constitutes the total thrust J of a propeller.

The parts near the extremities of the blades obviously contribute the greatest proportion of this total thrust, since j varies as the square of the circumferential velocity.[1] The useful work T_u of the propeller is measured (§ 83) by the product VJ of the forward speed multiplied by the thrust.

On the other hand, each portion of the blade offers a certain resistance to rotation q, all of which can be incorporated in the single resistance Q, applied at a certain distance l from the axis or, instead, by a moment of resistance of the value lQ.

The motive power T_m, absorbed by the propeller, is therefore expressed:

$$T_m = 2\pi n l Q,$$

n being the angular velocity in revolutions per second.

The ratio $\frac{T_u}{T_m}$ of the useful power to the motive power absorbed in obtaining the same gives the *total efficiency oj the propeller.*

It is really a mean of the efficiencies of the different portions of the blade, but not an arithmetical mean, since the work is principally done by the tips; and as the total value is not much greater than that of the efficiency of the blade-tips, the latter should be increased as much as possible — in fact, up to the maximum maximorum efficiency.

To realise this result, as shown in § 83, the actual angle of incidence of the portion of the blade must have its optimum value, and the inclination of the said portion

[1] It is supposed that the thrust j is the same as if the portion producing it acted by itself, which is scarcely probable. But as we are not here working to a strict formula, this may be admitted.

must be about 45°. The latter condition is hard to fulfil in practice, since, in starting, the actual angle of incidence of the tips, though less than the apparent angle, will be far greater than the optimum angle.

The propeller would thus work under the worst conditions, with the blades beating the air at too great an angle, and consequently absorbing power out of all proportion to the work done.[1]

However, the considerations which follow in § 87 show that the employment of such an inclination at the tips of the blades would only be justified if the speed of translation of the machine were very great and the rotary velocity of the propeller were small—that is to say, when the propellers are of very large diameter. Finally, any benefit accruing from adopting this method, from the standpoint of the total efficiency of the propeller, would not compensate for the difficulties of construction inseparable from it (especially in the case of wooden propellers).

It is a principle of construction—a *characteristic*—that two propellers, geometrically alike, have their tips inclined at the same angle, even though their diameters are not equal.

If the diameter of a propeller is D and its pitch H, the value of its inclination is $\frac{H}{\pi D}$.

This value is proportional to that of the ratio $h=\frac{H}{D}$ of the pitch to the diameter, known as the *pitch ratio* of the propeller.

The pitch ratio, in consequence, is a coefficient of form which remains the same for all propellers geometrically alike, and can therefore distinguish species of propellers.

[1] There are some systems of propellers whose pitch may be varied during flight at the will of the pilot, so that a small pitch can be used in starting and afterwards increased as the speed grows up to the desired inclination. Their employ—if they were practicable—would therefore facilitate starting.

At the present time the pitch ratio generally varies between 0·50 and 1; the latter value, corresponding to an inclination at the tips of about 17°, does not appear to have been exceeded in practice.

The efficiency of the tips in such propellers is not, therefore, maximum maximorum, and, as we have said, the total efficiency of a propeller is not much more than that of the tips.

In the Drzewiecki propeller, as all parts of the blade actually meet the air at the optimum angle, the efficiency —maximum at every point—goes on increasing from the tip to that portion inclined at about 45°, which is the maximum maximorum, and then decreases from that point onwards towards the boss. The total efficiency of the whole propeller, of course, always remains greater than that of the tips, though the influence on its value of the inner portions of the blade is quite small, owing to their slow rotational velocity.[1]

86. The fineness of a propeller.

The value i_1 of the optimum angle obviously influences that of the efficiency of the portion of the blade, and furthermore that of the total efficiency of the propeller. But this value depends on the suitability of the shape, on the polish of the surface—in a word, on the fineness of the propeller.

The greater the fineness, the smaller the optimum angle becomes, and the higher the maximum propeller efficiency.

It is therefore possible to write the fineness f as the inverse $\frac{1}{i_1}$ of the value of the optimum angle (as was done

[1] In the case of a propeller with a constant pitch, the variation of efficiency along the blade is not so easily followed, because the inclination and actual angle of incidence vary simultaneously and counteract each other.

in the case of the plane in § 14), and to regard it, like the pitch ratio, as a coefficient distinguishing a class of propellers. The characteristics of a propeller of constant pitch are, therefore, its diameter D, its pitch ratio *h*, and its fineness, taken as the inverse of the value of its optimum angle.

According to M. Drzewiecki, the value of the optimum angle for good propellers is about 0·03, which will lead, for the maximum efficiency of a portion of the blade, to a value greater than 0·9, and, for the total maximum efficiency of a variable-pitch propeller, to 0·9 or thereabouts.

Certain considerations lead one to suppose that these figures are a little exaggerated. In practice the value of the optimum angle is, for well-made propellers, about 0·075, and the total maximum efficiency of the best types of propellers yet constructed is about 75 per cent. (§ 19). One has a right, however, to hope that in the future this figure will be surpassed.

87. The relations between the characteristics of a propeller, its forward speed and its velocity of rotation.

Elsewhere it has been explained (§§ 19 and 29) that the efficiency of a propeller was maximum when a certain relation existed between its velocity of rotation and the speed of the machine propelled by it.

The reason of this should be apparent from statements occurring in foregoing sections.

For instance, we saw in § 85 that to get the highest efficiency out of a propeller, the actual angle of incidence of the tips of its blades should be the optimum angle; and in § 81 it was stated that the value of the actual angle of incidence of a portion of the blade of any given propeller depended on that of the relation $\frac{V}{n}$ between its forward speed and its rotational velocity. The actual angle

of incidence of a propeller tip cannot therefore be the optimum angle unless the ratio $\frac{V}{n}$ has one certain value.

As a result it can be definitely laid down that:

In order to get the highest efficiency out of any given propeller, there must be one certain ratio, and one only, between the rational velocity and the speed of the vehicle

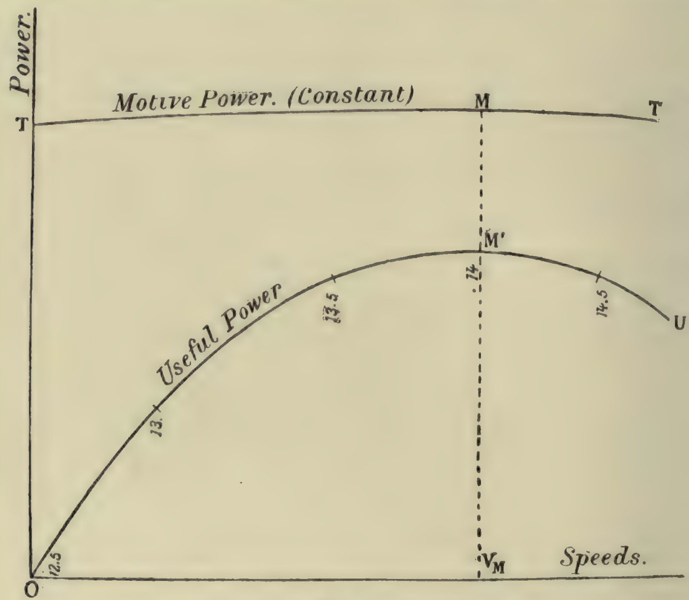

FIG. 88.

it propels, be it aeroplane, dirigible balloon, hydroplane, or what not.

This rule is extremely important, and dominates the entire problem of propulsion by the screw-propeller.

Its neglect has been a fruitful cause of error, and we have seen propellers built with great care giving most moderate results, for the simple reason that the speed of the machines they propel has no relation to their velocity of rotation.

However, it is possible to depart a little from a strict observance of this rule, since the maximum efficiency of a propeller varies but slightly, as will be seen by reference to Fig. 18 (reproduced as Fig. 88). Thus, when the cause is understood, the application of the rule is capable of accommodation to circumstances, while to misunderstand or to ignore it is to court failure.

In § 30 we stated that it was sometimes profitable to sacrifice a proportion of propeller efficiency, with a view to increasing the excess reserve of useful power; but, for convenience sake, *in all future statements the propeller will be taken as giving out its maximum efficiency.*

The value of the relation that should exist between the speed of rotation n of a propeller and the forward speed V_m—the *good speed* suited to this velocity of rotation—depends, obviously, on the value of the propeller's characteristics; and we arrive at the idea of what this relation should be when we say that the actual angle of incidence of the tip of the blade (the value of which depends on those of the characteristics D and h—the diameter and the pitch ratio respectively) should be equal to the optimum angle i_1, or the third characteristic of the propeller.

By expressing this equality algebraically, an extremely simple formula is obtained, giving the relation [1] between all the above quantities, as follows:

$$V_m = anD, \tag{26}$$

[1] The result is obtained in the following way. The actual angle of incidence i of a portion of the blade is equal (§ 81) to the difference between the two angles I and I′, the trigonometrical tangents of which have for their respective values $\frac{H}{\pi d}$ or, what is equivalent, $\frac{h}{\pi}$ and $\frac{V}{\pi n d}$. Assuming the tips of the blade at a distance $\frac{D}{2}$ from the axis, to be the portion in question, the equality between the actual angle of incidence and the optimum angle is expressed by:

$$i_1 = I - I',$$

which leads to the following:

$$\tan i_1 = \frac{\tan I - \tan I'}{1 + \tan I \tan I'}.$$

a being a coefficient the value of which is given by the formula:

$$(27) \qquad a = h - \pi\,(1 + 0{\cdot}1\,h^2)i_1 .$$

Formula (26) is of the utmost importance. It clearly shows that for propellers of the same type, that is, with the same fineness and pitch ratio—in which case according to formula (27) the coefficient a is constant—the good forward speed V_m is proportional to the product nD, which is itself proportional to the rotational velocity πnD of the tip of the blade. Therefore:

In order that a propeller of a given type should give out its maximum efficiency, the speed of the machine which it propels should be a definite fraction of the rotational velocity of the tips of the blades.

This fraction is expressed in the term $\frac{a}{\pi}$, and its value increases with that of the coefficient a, that is to say, as the pitch ratio of the propeller is increased.

If the optimum angle i_1 is given the value 0·075 (see § 86), the value of the coefficient a can be calculated as a function of h, the value of the pitch ratio.

The table below gives for this coefficient seven theoretical values, of which the first six correspond to the usual values of the pitch ratio.

The angle i_1 being small, i_1 can be substituted for tan i_1, and $\tan^2 I$ for tan I tan I′. Then giving the tangents their respective values, and taking into consideration that the speed V has assumed its good value V_m, the following is obtained:

$$i_1 = \frac{\frac{h}{\pi} - \frac{V_m}{\pi n D}}{1 + \frac{h^2}{\pi^2}} .$$

Or, solving for V_m and taking π^2 as equal to 10:

$$V_m = [h - \pi\,(1 + 0{\cdot}1\,h^2)\,i_1]\,nD ,$$

which is the same as formula (26), in which the value of a given in formula (27) has been substituted for the coefficient itself.

Pitch ratio h . . (Ratio of the pitch to the diameter of the propeller.)	0·75	0·80	0·85	0·90	0·95	1	π
Coefficient a . .	0·5	0·55	0·60	0·65	0·69	0·74	2·67

Most of the present day propellers have a pitch ratio of about 0·75, and, for such, formula (26) becomes:

$$(26a) \qquad V_m = 0{\cdot}5\, nD.$$

This shows that the forward speed of such propellers should be a fraction $\frac{a}{\pi} = \frac{0{\cdot}5}{3{\cdot}14}$ or slightly less than $\frac{1}{6}$th of the rotational velocity of the tip of the blade.

There is a tendency among manufacturers to increase the pitch ratio up to 0·85, 0·90, and even to 1. In this last case the value of a would be 0·74, and the good speed should be a little less than $\frac{1}{4}$th of the rotational velocity of the tip of the blade.

In the case (see § 85) where the inclination of the tip of the blade is 45°, and consequently the efficiency of this portion is maximum maximorum, $h = \pi$, and the coefficient $a = 2{\cdot}67$.

Formula (26) then becomes:

$$(26b) \qquad V_m = 2{\cdot}67\, nD.$$

An idea of the result of using such a pitch ratio may be obtained by assuming that it is required to fit a propeller of this type to an aeroplane which should move at the speed of 26·7 metres per second. Formula (26*b*) gives the relation $nD = 10$. The rotational velocity of the tips of the blades, in order that the propeller may work under the best conditions, ought then to be $\pi nD = 31{\cdot}4$ m.p.s. This velocity is extremely low in comparison with that which at present obtains, which frequently is more than

150 m.p.s.[1] In the case just taken it would be necessary to use very large propellers to produce an appreciable thrust or perhaps to use several (see § 90), but it is quite true that an increased efficiency is reached thereby. However, reasons of a practical order against the use of propellers with so great a pitch ratio—such as difficulty in starting and construction, &c.—have already been mentioned in § 85.

88. Calculation of an aeroplane propeller.

The complete theory of the screw-propeller shows that the thrust J produced by a propeller of diameter D advancing at a *good speed* V_m is given by the simple formula:

$$J = bV_m^2D^2, \tag{28}$$

where b stands for a coefficient whose value—as in the case of the coefficient a in formula (26)—depends on that of the characteristics h and i_1 of the propeller.

It would take too long to examine the variation of the value of this coefficient b with that of the characteristics mentioned above. Suffice it to state that in the case where the pitch ratio is 0·75, the optimum angle always being assumed as equal to 0·075, the coefficient b has the *theoretical* value 0·045.

By the use of formulæ (26) and (28) it is very easy to calculate the diameter and rotational velocity of a propeller of a given species (characterised by the values of coefficients a and b) suitable for propelling a given aeroplane or even, to take a more general case, any kind of vehicle.

The known factors of the problem are the speed V_m, at which the vehicle must be propelled, and the thrust J required to propel it at the speed V_m:

Applying formula (28), we obtain the value of the diameter of a suitable propeller, that is, of a propeller

[1] With wooden propellers.

which, advancing at the good speed V_m, is capable of producing the required thrust J:

$$D=\sqrt{\frac{J}{bV_m^2}}$$

Formula (26), again, gives the value of the angular velocity at which the propeller should rotate:

$$n=\frac{V_m}{aD}.$$

In the case of an aeroplane the problem may be set as follows:

Calculate the diameter and rotational velocity of a propeller of a given type (characterised by the values of coefficients a and b) capable of propelling at a speed V_m *an aeroplane of given weight* P, *plane area* S, *and fineness f (the lifting efficiency* K *of the planes being assumed equal to* 0·4).

In order that the problem may be capable of solution, the first condition is that the speed V_m must be one of the attainable speeds (see § 8) of the aeroplane; that is, it must correspond to an admissible and safe value of the angle of incidence. If this condition is fulfilled, it is possible, by the use of Tables I., II., and III. (§§ 10 and 17) or of the relation (14) set out in § 26 (and reproduced again in § 89), to calculate the value of the thrust required to sustain an aeroplane at the speed V_m.

An example will render the exquisite simplicity of this method of calculation even clearer.

The aeroplane of fineness $\frac{1}{0{\cdot}07}$, weighing 500 kg., and of 50 sq. m. plane area, is to be propelled at a speed of 17·68 m. p. sec. What are to be the diameters and rotational velocity of a propeller of average type (pitch ratio 0·75) suitable to be fitted to this aeroplane?

In the first place, Table I. shows that the speed 17·68 m.

p. sec. is, in fact, an attainable speed for the aeroplane in question, the loading of which is 10 kg. per sq. metre. This speed corresponds to the angle of incidence 0·08.

Secondly, by the aid of Table III. it is possible to calculate the value of the thrust required to sustain the aeroplane at the above angle of incidence. This value is 70·6 kg.

In the case of propellers of the usual type employed at the present time, formulæ (26) and (28) become, as already stated:

$$(26a) \qquad V_m = 0{\cdot}5\, n\mathrm{D}.$$
$$(28a) \qquad \mathrm{J} = 0{\cdot}045\, V_m^2 \mathrm{D}^2.$$

By applying formula (28a) we obtain as the value of the diameter required:

$$\mathrm{D} = \sqrt{\frac{70{\cdot}6}{0{\cdot}045 \times (17{\cdot}68)^2}} = \text{about } 2{\cdot}243 \text{ m.}$$

And the rotational velocity of the propeller, therefore, assumes the value, according to formula (26a):

$$n = \frac{17{\cdot}68}{0{\cdot}5 \times 2{\cdot}243} = \text{about } 15{\cdot}8 \text{ r. p. sec., or } 948 \text{ r.p.m.}$$

Thus the propeller suitable for driving the aeroplane in question should have a diameter of about 2·25 m. and revolve at about 950 r.p.m.

It should, however, be observed that formulæ (26a) and (28a), though perfectly correct for a given type of propeller, may lead to results that are impossible in practice. In particular, if the speed of flight to be attained is high, 25 m. p. sec. for instance, the value of the product $n\mathrm{D}$ derived from formula (26a) would be 50, corresponding to a peripheral velocity of 157 m. p. sec., which may well appear excessive.

The means to be adopted in such a case is to employ a different type of propeller with a greater pitch ratio—

0·90 for instance. This would give the coefficient a, in formula (26), a value 0·65, and the product nD would have the more reasonable value 38.

Similarly, it may be necessary to use a certain engine which runs at a given number of revolutions when developing its full power. If the rotational velocity n resulting from formula (26a) differs widely from the number of revolutions of the engine, it becomes necessary either to gear down the propeller or to modify its pitch ratio.[1]

Up to now nothing has been said of the calculation of the motive power to be exerted on the propeller shaft. This power may be easily obtained through dividing the useful power $V_m J$ by the efficiency of the propeller, the value of which depends on that of its characteristics, the pitch ratio and the fineness. Since the values of V_m and J are given respectively by formulæ (26) and (28), the value of the motive power is obtained from the following:

$$(29) \qquad T_m = cV_m^3 D^2,$$

wherein the value of the coefficient c depends, as in the case of a and b, on the value of the propeller's characteristics.

With good propellers of the usual type, whose efficiency is about 70 per cent., formula (29) becomes (T_m being expressed in H.P.):

$$(29a) \qquad T_m = 0{\cdot}00086 V_m^3 D^2.$$

Applying formula (29a) to the example given above, gives, as the value of the motive power required to sustain the aeroplane, 23·8 H.P., which is less than the 33·3 H.P. that would have been obtained from Tables II. and IV. (§ 24). This divergence is due to the fact that Table IV.

[1] The value of coefficients a and b corresponding to the various values of the pitch ratio may be found theoretically, or, with greater certainty, by practical experiment. Such practical experiments would prove of the very highest value to constructors.

was based on the *average* value obtained in practice with the propellers of present-day aeroplanes. This average value cannot be reckoned as 70 per cent., firstly, because only a limited number of propellers attain to this degree of efficiency, and secondly, because the propeller is not always working under the most favourable conditions. The *mean* efficiency on which Table IV. was based was reckoned as 50 per cent.

89. Relation between the propeller diameter and the detrimental surface of the aeroplane.

The known factors in the general problem just considered were the speed of the vehicle to be propelled and the thrust required to attain this speed.

Now these two factors, so far from being independent of one another, are, in the case of an aeroplane, as has been seen in § 26, connected by the following relation:

$$(14) \qquad t = \frac{P^2}{KSV^2} + 0{\cdot}08sV^2,$$

where t, P, K, S, and s stand respectively for the thrust required for sustentation, the speed, the weight, the lifting efficiency, the plane area, and the detrimental surface of the aeroplane.

Thus, in calculating the most suitable propeller for a given aeroplane, the speed cannot be arbitrarily fixed at the same time as the thrust required to obtain this speed, since the value of the one determines the other.

Referring now to what was said in § 88, it will be seen that the method of calculation therein set out leads to the expression, the speed under consideration being a good speed:

$$(28) \qquad J = bV^2D^2.$$

Since equations (14) and (28) both express the value of the same quantity—the thrust required to sustain the

aeroplane and the propeller thrust that produces this sustentation—their second members may be equated:

$$bV^2D^2 = \frac{P^2}{KSV^2} + 0{\cdot}08sV^2,$$

or, extracting the speed:

$$V^4 = \frac{P^2}{KS(bD^2 - 0{\cdot}08s)}. \tag{30}$$

The important feature of this formula resides in the fact that, if V is to have real and not infinite values, the expression within the brackets in the denominator must necessarily be positive; in other words, $bD^2 > 0{\cdot}08s$, or:

$$D > \sqrt{\frac{0{\cdot}08s}{b}}. \tag{31}$$

Hence, in order that one of the speeds at which the aeroplane can travel may be a *good speed* so far as the propeller efficiency is concerned, the diameter of the propeller must not be less than a certain limit, the value of which depends on that of the detrimental surface of the aeroplane. Consequently:

Larger propellers must be used, the larger the detrimental surface of the aeroplane; and the propeller diameter must not be less than a certain limit, otherwise the efficiency drops.

This feature deserves consideration. It is certainly remarkable that the diameter in no way depends on the value of the other characteristics—the weight and plane area of the aeroplane.

It amounts to this, that there must exist a fixed ratio between the surface of the circle swept by the propeller and the imaginary disc (§ 11) that represents the detrimental surface. It could easily be shown that, with the usual type of propeller with a pitch ratio of 0·75, the propeller diameter must be greater than the product of the diameter of the imaginary disc aforesaid by 1·4.

Example

The detrimental surface of the aeroplane being 1·80 sq. metres, what is the smallest diameter for a propeller of the usual type with a pitch ratio of 0·75 ?

The value of the diameter required, according to (31) is:

$$\sqrt{\frac{0{\cdot}08 \times 1{\cdot}80}{0{\cdot}045}} = \text{about } 1{\cdot}79 \text{ metres.}$$

In practice, as a matter of fact, this limit for the diameter must be exceeded,[1] since it corresponds, as shown by formula (30), to an infinite value of the good speed. But, on the other hand, the dimension of the diameter should not be exaggerated, as this would lead to too small a value of good speed.

In any case, by following the method of calculation set out in § 88, one can arrive with sufficient accuracy at the correct diameter for a given speed, without incurring the danger of falling into either excess.

90. The value of static tests.

From the foregoing it is clear that the one quality to be sought for in a propeller is a high maximum efficiency, the value of which depends on the pitch ratio and the fineness.

The only method of measuring directly by experiment the value of the maximum efficiency of a propeller is to cause it to propel some kind of vehicle and to measure at different speeds the thrust exerted in travelling as well as the motive power absorbed. This method of testing, however, requires complicated apparatus, that has only as yet been installed in a very few laboratories. The

[1] Since the value depends on that of coefficient b, it varies with the pitch ratio of the propeller. If the latter is greater than 0·75, the diameter limit will also be greater.

majority of constructors are therefore content with testing the propeller statically, or on the bench; that is, they measure, at different velocities of rotation, the thrust it exerts and the motive power required to produce this thrust.

Tests of this nature cannot, however, as a rule furnish direct evidence of the value of propeller efficiency.[1] They only afford information regarding the capacity of the propeller to produce, under analogous conditions of working—that is, statically—the greatest possible thrust for a given expenditure of motive power; in other words, they afford information regarding the quality of the propeller *as a lifting-screw.* The term *quality* is here purposely employed, as it was used by Colonel Renard to designate a coefficient which defines the common value, as lifting-screws, of propellers belonging to the same species, that is, possessing a similar geometrical shape.

The thrust J (in kilogrammes) given out on the bench by a propeller of diameter D (in metres) driven by a motive power T_m (in H.P.) satisfies, as shown experimentally by Colonel Renard, the relation:

$$(32) \qquad J^3 = AT_m^2D^2,$$

where A is a coefficient that measures the quality of the propeller as a lifting-screw, a quality that only depends on the shape of the propeller, that is, on its finenes $\frac{1}{i_1}$ (§ 86) and, if the pitch is constant, on its pitch ratio h.

For the best lifting-screws known the value of this quality A varies between 450 and 500; in some cases it may even reach 550.

An example will show more clearly the practical significance of formula (32).

[1] Captain Ferber has, however, shown that the coefficients of certain formulæ applicable to the study of screw-propellers could be calculated from the basis of static tests.

Calculate the thrust produced, statically, by a propeller of 2 m. diameter, of the type defined by its value of A = 480, driven by 10 h.p.

Applying formula (32), we obtain:

$$J^3 = 480 \times 100 \times 4,$$

therefore $J = 57{\cdot}69$ kg.

Thus, to test a propeller statically, we must not calculate the simple ratio of thrust to power, for this ratio is in fact variable with the rotational velocity and diameter of the propeller.

We have to find the value of $A = \frac{J^3}{T_m^2 D^2}$, which is approximately constant, whatever the rotational velocity or the diameter. The propeller is a better lifting-screw, the greater the above value (which should be about 480 if the propeller is to be of normal quality).

Nor is a single test sufficient, for its results may easily be subject to error; the propeller must be rotated at different powers, and the mean taken of all the results in order to form an exact idea of the value of its quality.

The quality A is therefore the final test of the value of a lifting-screw. The same screw used as a propeller under the best conditions would have a maximum efficiency e. One would be naturally inclined to believe that these two qualities A and e vary one with the other, and that, of two propellers tested statically, the one with the best lifting quality would give the highest maximum efficiency when used for propulsion.

But this is not so, save in the one case where the propellers to be compared (assuming the pitch to be constant) have the same pitch ratio; for in this case the lifting quality and maximum efficiency depend only on the fineness of the propeller and vary with it.

But, generally speaking, the lifting quality and the maximum propelling efficiency vary principally with the

value of the pitch ratio (the pitch being assumed constant[1]), and not necessarily in the same sense. Thus, it has already been seen (§ 85) that the efficiency of a propeller is maximum maximorum when the value of its pitch ratio is somewhere about π, that is, 3·14.

Now, Colonel Renard has shown by experiment that the quality of a lifting-screw is highest when its pitch ratio has a value of about 0·75.

Consequently, if we compared a series of screws of pitch ratios ranging from 0·75 to 3·14, we should find that their propelling efficiency grows with the value of the pitch ratio, while their lifting quality diminishes. For instance, a screw with a pitch ratio of 2·5 would, if used as a propeller under good conditions (§ 87), have excellent efficiency, while it would be an indifferent lifting-screw.

As a matter of fact, the average type of screw-propeller used at the present day has a pitch ratio of about 0·75, so that it is, as a rule, a good lifting-screw at the same time. This, however, is a pure coincidence. If we suppose that in the future propellers of much greater pitch ratio than 0·75 will be used to propel high-speed aeroplanes, such propellers would probably give far worse results when tested statically than those at present in use.

91. Tilt of the aeroplane due to propeller action—Gyroscopic effect—Use of two propellers.

If the aeroplane possessed no automatic lateral stability, and if its axis of rotation (§ 58) coincided with the propeller axis, the revolution of the propeller would cause the aeroplane to rotate slowly in the opposite direction. A tendency to this effect is actually noticeable in reality,

[1] In the case of variable-pitch propellers (such as those designed by Drzewiecki for instance) the question becomes extremely complicated; in any case it has not yet been proved that the same helicoidal surfaces are suitable for both screw-propellers and lifting-screws.

but it is checked, if the aeroplane is laterally stable, by the righting moment that arises from its tilt.

The aeroplane then flies with a slight permanent lateral tilt, which produces a constant tendency to turn it. Since such a tilt cannot be permitted in practice, two methods may be adopted to prevent it: either the position of the centre of gravity must be rendered unsymmetrical by suitably distributing the masses in the aeroplane, or two propellers must be used turning in opposite directions, thus counter-balancing one another.

Consideration of another effect, known as the gyroscopic effect, may also lead to the adoption of the latter method.

It is known that when a force is applied to the axis of a gyroscope tending to displace it in a certain direction, it actually tends to move in a direction at right angles to the former, and with the greater energy according as the force applied to it is more violent, the velocity of revolution higher, and the moment of inertia of the gyroscope greater.

The propeller of an aeroplane may set up effects of this kind, more especially if it is heavy; thus, a sudden horizontal deviation may, if violent enough, set up a tendency to pitch longitudinally.

This tendency, undesirable even in itself, has the further disadvantage of subjecting the framework to stresses which may easily become dangerous if they have not been provided for. Consequently, it is desirable to eliminate the gyroscopic effect, which can be done by the use of two propellers; but, although this method has several advantages, there are distinct practical difficulties that have hitherto strictly limited its use.

The calculation of two propellers follows precisely the lines of the method used in the case of a single one. We only need to apply formula (28), giving the thrust J half the value of the thrust required to sustain the aeroplane.

From this we can deduce the required propeller diameter and, applying formula (26), the joint rotational velocity.

Referring once again, for the sake of greater clearness, to the example already considered in § 88, where the required thrust was 70·6 kg., we should require in the present case a thrust of 35·3 kg. from each propeller, which gives the diameter:

$$D=\sqrt{\frac{35 \cdot 3}{0 \cdot 045\times(17 \cdot 68)^2}}=\text{about } 1 \cdot 58 \text{ m.}$$

The velocity of rotation is:

$$n=\frac{17 \cdot 68}{0 \cdot 5\times 1 \cdot 58}=22 \cdot 38 \text{ r.p.s., or } 1349 \text{ r.p.m.}$$

These brief calculations show that we can apply the results for a single propeller to the case of two propellers by multiplying the number of revolutions and dividing the diameter by $\sqrt{2}$, or 1·414.

The power required for sustentation does not vary,[1] since it is proportional to the product of the given factors of the problem—the speed of the aeroplane and the thrust required to attain this speed.

92. Influence of the curve and shape of propeller-blades—Influence of the number of blades.

In considering the planes of the aeroplane, the important effect of the curve in the planes was examined, both in increasing the lift and diminishing the drift. It seems only reasonable to suppose that the curvature of the propeller-blades should conduce to similar effects, and that, in the case of screws used for propulsion, these advantages should be evidenced in an increased efficiency.

Nothing absolutely definite is known in regard to this point, but various considerations would seem to show that

[1] Provided, of course, that all the propellers in question have, as we have tacitly assumed, the same maximum efficiency.

one of the chief effects of curving the propeller-blade is to maintain the efficiency to a better extent when the actual angle of incidence of the blade, that is, the ratio between the forward speed and the rotational velocity, varies. In other words, this constructional device would enable the propeller to be used at speeds differing considerably from a good speed, without incurring a serious loss of efficiency.

The shape in plan-form of the propeller-blade must obviously affect the maximum efficiency of the propeller; but on this point few experiments have been made, and guesswork is not wholly absent from design in this respect.

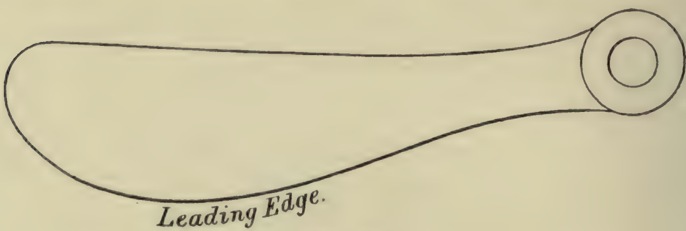

FIG. 89.

Many constructors design their propellers with a curved leading edge (Fig. 89). It has been stated by some, in support of this conformation, that the marks left on the propeller-blade, after rotation, by dust particles and oil assume this shape. Others—and with better apparent justification—claim that the curved leading edge reduces the vibration due to the torsional stresses on the blades.

Finally it may be stated that:

An increase in the number of blades only seems to have a slight effect on the efficiency of a propeller.

It appears, however, that a four-bladed propeller is slightly more efficient than one with two blades.

93. Bending stress on a propeller-blade — Articulated blades.

The thrust and the resistance to rotation exerted on

a propeller-blade, which is only maintained in position by being fixed in the boss, subject it to a bending stress; such forces may deform or even break it, if its strength is insufficient, and in any case probably constitute a source of harmful vibration.

Colonel Renard invented an ingenious method of overcoming this disadvantage without increasing the weight

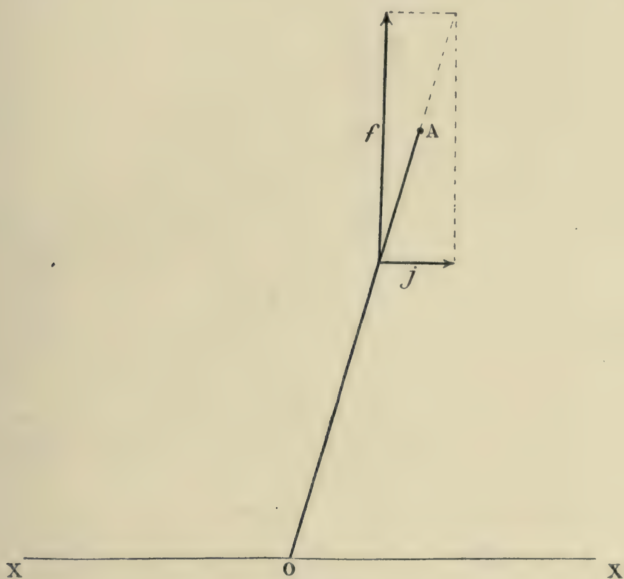

FIG. 90.

of the propeller. This method consists in articulating the blades at the point where they are mounted on the boss.

Let OA (Fig. 90) represent diagrammatically a propeller blade articulated at O.

Since the blade is subjected on the one hand to a thrust j, and on the other to a centrifugal force f, it automatically takes the direction of the resultant of the two

forces exerted upon it, and is consequently no longer subjected to a bending strain.

Similarly, by articulating the blade in the direction in which is exerted the resistance to rotation, or drift, q (Fig. 91), this force is neutralised by the centrifugal force, and the blade axis BA takes the direction of the resultant. But for this, the point of articulation B must be situated at some distance from the propeller axis XX.

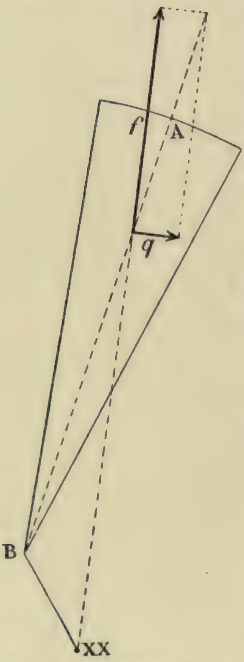

FIG. 91.—Elevation, perpendicular to axis.

It should, however, be noted that the direction taken, in both cases under notice, by the propeller-blade is independent of the rotational velocity, since each of the three forces j, q, and f is proportional to the square of this velocity.

To apply Colonel Renard's method each blade must be doubly articulated, with a Cardan joint for instance.

Propellers of this type have, in fact, been built.[1] But, as Colonel Renard himself showed, it may be sufficient for practical purposes to give the blade its suitable direction in both senses, the while keeping the propeller rigid. By this means the bending strain would be eliminated.

Throughout the course of the foregoing treatise every effort has been made to preserve its theoretical nature; for this reason care has been taken not to overload it with

[1] The propellers of the Italian dirigible, for instance. Other propellers (German "Parseval" type) are limp when at rest, and only acquire the necessary rigidity, when rotating, by centrifugal force.

such extraneous matter as historical references, descriptions of machines, or details of construction.

Recourse has only been had to the most elementary principles of mathematics and mechanics, while an attempt has been made to reduce to the lowest possible point the use of formulæ, the mere sight of which only too often inclines the reader to throw aside the book that contains them, after a rapid glance at its pages. Perhaps it is not too much to hope that the present work at first sight will not appear too dry and severe, in spite of the strictly scientific nature of its subject, to find a place upon those library shelves that are reserved for books of enduring interest and value.

And, on the other hand, some of its readers—who might well have been repelled at first by books of a more scientific character—may be led to take a deeper interest in the subject whereof it treats, and so be induced to make a more profound study, in more advanced works, of many problems that had perforce to be dismissed in somewhat summary fashion in the foregoing pages.

Even had we attained to no better result than this, to serve as it were as an introduction to scientific authority of greater eminence, we would deem ourselves fortunate in having been able to assist, however slightly, the furtherance of the grandest task whereto man has ever set his hand; in having contributed a pebble to the magnificent edifice that future generations will behold complete.

TABLE OF SQUARES

NUMBERS	SQUARES	NUMBERS	SQUARES	NUMBERS	SQUARES
10·0	100·00	15·4	237·16	20·8	432·64
10·1	102·01	15·5	240·25	20·9	436·81
10·2	104·04	15·6	243·36	21·0	441·00
10·3	106·09	15·7	246·49	21·1	445·21
10·4	108·16	15·8	249·64	21·2	449·44
10·5	110·25	15·9	252·81	21·3	453·69
10·6	112·36	16·0	256·00	21·4	457·96
10·7	114·49	16·1	259·21	21·5	462·25
10·8	116·64	16·2	262·44	21·6	466·56
10·9	118·81	16·3	265·69	21·7	470·89
11·0	121·00	16·4	268·96	21·8	475·24
11·1	123·21	16·5	272·25	21·9	479·61
11·2	125·44	16·6	275·56	22·0	484·00
11·3	127·69	16·7	278·89	22·1	488·41
11·4	129·96	16·8	282·24	22·2	492·84
11·5	132·25	16·9	285·61	22·3	497·29
11·6	134·56	17·0	289·00	22·4	501·76
11·7	136·89	17·1	292·41	22·5	506·25
11·8	139·24	17·2	295·84	22·6	510·76
11·9	141·61	17·3	299·29	22·7	515·29
12·0	144·00	17·4	302·76	22·8	519·84
12·1	146·41	17·5	306·25	22·9	524·41
12·2	148·84	17·6	309·76	23·0	529·00
12·3	151·29	17·7	313·29	23·1	533·61
12·4	153·76	17·8	316·84	23·2	538·24
12·5	156·25	17·9	320·11	23·3	542·89
12·6	158·76	18·0	324·00	23·4	547·56
12·7	161·29	18·1	327·61	23·5	552·25
12·8	163·84	18·2	331·24	23·6	556·96
12·9	166·41	18·3	334·89	23·7	561·69
13·0	169·00	18·4	338·56	23·8	566·44
13·1	171·61	18·5	342·25	23·9	571·21
13·2	174·24	18·6	345·96	24·0	576·00
13·3	176·89	18·7	349·69	24·1	580·81
13·4	179·56	18·8	353·44	24·2	585·64
13·5	182·25	18·9	357·21	24·3	590·49
13·6	184·96	19·0	361·00	24·4	595·36
13·7	187·69	19·1	364·81	24·5	600·25
13·8	190·44	19·2	368·64	24·6	605·16
13·9	193·21	19·3	372·49	24·7	610·09
14·0	196·00	19·4	376·36	24·8	615·04
14·1	198·81	19·5	380·25	24·9	620·01
14·2	201·64	19·6	384·16	25·0	625·00
14·3	204·49	19·7	388·09	25·1	630·01
14·4	207·36	19·8	392·04	25·2	635·04
14·5	210·25	19·9	396·01	25·3	640·09
14·6	213·16	20·0	400·00	25·4	645·16
14·7	216·09	20·1	404·01	25·5	650·25
14·8	219·04	20·2	408·04	25·6	655·36
14·9	222·01	20·3	412·09	25·7	660·49
15·0	225·00	20·4	416·16	25·8	665·64
15·1	228·01	20·5	420·25	25·9	670·81
15·2	231·04	20·6	424·36	26·0	676·00
15·3	234·09	20·7	428·49	26·1	681·21

Table of Squares (*continued*)

NUMBERS	SQUARES	NUMBERS	SQUARES	NUMBERS	SQUARES
26·2	686·44	31·6	998·56	37·0	1 369·00
26·3	691·69	31·7	1 004·89	37·1	1 376·41
26·4	696·96	31·8	1 011·24	37·2	1 383·84
26·5	702·25	31·9	1 017·61	37·3	1 391·29
26·6	707·56	32·0	1 024·00	37·4	1 398·76
26·7	712·89	32·1	1 030·41	37·5	1 406·25
26·8	718·24	32·2	1 036·84	37·6	1 413·76
26·9	723·61	32·3	1 043·29	37·7	1 421·29
27·0	729·00	32·4	1 049·76	37·8	1 428·84
27·1	734·41	32·5	1 056·25	37·9	1 436·41
27·2	739·84	32·6	1 062·76	38·0	1 444·00
27·3	745·29	32·7	1 069·29	38·1	1 451·61
27·4	750·76	32·8	1 075·84	38·2	1 459·24
27·5	756·25	32·9	1 082·41	38·3	1 466·89
27·6	761·76	33·0	1 089·00	38·4	1 474·56
27·7	767·29	33·1	1 095·61	38·5	1 482·25
27·8	772·84	33·2	1 102·24	38·6	1 489·96
27·9	778·41	33·3	1 108·89	38·7	1 497·69
28·0	784·00	33·4	1 115·56	38·8	1 505·44
28·1	789·61	33·5	1 122·25	38·9	1 513·21
28·2	795·24	33·6	1 128·96	39·0	1 521·00
28·3	800·89	33·7	1 135·69	39·1	1 528·81
28·4	806·56	33·8	1 142·44	39·2	1 536·64
28·5	812·25	33·9	1 149·21	39·3	1 544·49
28·6	817·96	34·0	1 156·00	39·4	1 552·36
28·7	823·69	34·1	1 162·81	39·5	1 560·25
28·8	829·44	34·2	1 169·64	39·6	1 568·16
28·9	835·21	34·3	1 176·49	39·7	1 576·09
29·0	841·00	34·4	1 183·36	39·8	1 584·04
29·1	846·81	34·5	1 190·25	39·9	1 592·01
29·2	852·64	34·6	1 197·16	40·0	1 600·00
29·3	858·49	34·7	1 204·09	40·1	1 608·01
29·4	864·36	34·8	1 211·04	40·2	1 616·04
29·5	870·25	34·9	1 218·01	40·3	1 624·09
29·6	876·16	35·0	1 225·00	40·4	1 632·16
29·7	882·09	35·1	1 232·01	40·5	1 640·25
29·8	888·04	35·2	1 239·04	40·6	1 648·36
29·9	894·01	35·3	1 246·09	40·7	1 656·49
30·0	900·00	35·4	1 253·16	40·8	1 664·64
30·1	906·01	35·5	1 260·25	40·9	1 672·91
30·2	912·04	35·6	1 267·36	41·0	1 681·00
30·3	918·09	35·7	1 274·49	41·1	1 689·21
30·4	924·16	35·8	1 281·64	41·2	1 697·44
30·5	930·25	35·9	1 288·81	41·3	1 705·69
30·6	936·36	36·0	1 296·00	41·4	1 713·96
30·7	942·49	36·1	1 303·21	41·5	1 722·25
30·8	948·64	36·2	1 310·44	41·6	1 730·56
30·9	954·81	36·3	1 317·69	41·7	1 738·89
31·0	961·00	36·4	1 324·96	41·8	1 747·24
31·1	967·21	36·5	1 332·25	41·9	1 755·61
31·2	973·44	36·6	1 339·56	42·0	1 764·00
31·3	979·69	36·7	1 346·89	42·1	1 772·41
31·4	985·96	36·8	1 354·24	42·2	1 780·84
31·5	992·25	36·9	1 361·61	42·3	1 789·29

TABLE OF SQUARES (*continued*)

NUMBERS	SQUARES	NUMBERS	SQUARES	NUMBERS	SQUARES
42·4	1 797·76	43·3	1 874·89	44·2	1 953·64
42·5	1 806·25	43·4	1 883·56	44·3	1 962·49
42·6	1 814·76	43·5	1 892·25	44·4	1 971·36
42·7	1 823·29	43·6	1 900·96	44·5	1 980·25
42·8	1 831·84	43·7	1 909·69	44·6	1 989·16
42·9	1 840·41	43·8	1 918·44	44·7	1 998·09
43·0	1 849·00	43·9	1 927·21	44·8	2 007·04
43·1	1 857·61	44·0	1 936·00	44·9	2 016·01
43·2	1 866·24	44·1	1 944·81	45·0	2 025·00

TABLE OF SQUARE ROOTS

NUMBERS	SQUARE ROOTS	NUMBERS	SQUARE ROOTS	NUMBERS	SQUARE ROOTS
5·5	2·345	20·5	4·528	35·5	5·958
6·0	2·449	21·0	4·583	36·0	6·000
6·5	2·550	21·5	4·637	36·5	6·042
7·0	2·646	22·0	4·690	37·0	6·083
7·5	2·739	22·5	4·743	37·5	6·124
8·0	2·828	23·0	4·796	38·0	6·164
8·5	2·915	23·5	4·848	38·5	6·205
9·0	3·000	24·0	4·890	39·0	6·245
9·5	3·082	24·5	4·950	39·5	6·285
10·0	3·162	25·0	5·000	40·0	6·325
10·5	3·240	25·5	5·050	40·5	6·364
11·0	3·317	26·0	5·099	41·0	6·403
11·5	3·391	26·5	5·148	41·5	6·442
12·0	3·464	27·0	5·196	42·0	6·481
12·5	3·536	27·5	5·244	42·5	6·519
13·0	3·606	28·0	5·291	43·0	6·557
13·5	3·674	28·5	5·339	43·5	6·595
14·0	3·742	29·0	5·385	44·0	6·633
14·5	3·808	29·5	5·431	44·5	6·671
15·0	3·873	30·0	5·477	45·0	6·708
15·5	3·937	30·5	5·522	45·5	6·745
16·0	4·000	31·0	5·568	46·0	6·782
16·5	4·062	31·5	5·612	46·5	6·819
17·0	4·123	32·0	5·657	47·0	6·856
17·5	4·183	32·5	5·701	47·5	6·892
18·0	4·243	33·0	5·745	48·0	6·928
18·5	4·301	33·5	5·788	48·5	6·964
19·0	4·359	34·0	5·831	49·0	7·000
19·5	4·416	34·5	5·874	49·5	7·036
20·0	4·472	35·0	5·916	50·0	7·071

METRIC AND ENGLISH EQUIVALENTS

Linear

1 mm.	=	0·03937	inches	1 inch	=	25·4	mm.
1 cm.	=	0·3937	„	„	=	2·54	cm.
1 m.	=	39·37	„	„	=	0·0254	m.
„	=	3·2809	feet	1 foot	=	0·3048	m.
1 km.	=	1093·63	yards	1 mile	=	1609·3	m.
„	=	3280·9	feet		=	1·609	km.
„	=	0·6214	mile				

Square

1 sq. cm.	=	0·155	sq. inch	1 sq. inch	=	6·452	sq. cm.
„	=	0·001076	sq. feet	„	=	0·000645	sq. m.
1 sq. m.	=	10·764	sq. feet	1 sq. foot	=	0·0929	„

Cubic

1 cu. m.	=	35·32	cu. ft.	1 cu. foot	=	0·0283	cu. m.

Weight

1 gramme	=	0·0353	oz.	1 oz.	=	28·35	gram.
„	=	0·0022	lb.	„	=	0·02835	kg.
1 kg.	=	2·205	„	1 lb.	=	0·454	kg.

TABLE I

Feet and Metre Conversion Table

Feet.		Metres.
3·281	1	·305
6·562	2	·609
9·843	3	·914
13·123	4	1·219
16·404	5	1·524
19·68	6	1·829
22·96	7	2·134
26·25	8	2·438
29·52	9	2·743
32·81	10	3·047
36·09	11	3·353
39·37	12	3·657
42·65	13	3·962
45·93	14	4·267
49·21	15	4·572
52·49	16	4·877
55·77	17	5·181
59·06	18	5·486
62·34	19	5·795
65·62	20	6·095
98·43	30	9·144
131·23	40	12·192
164·04	50	15·240
196·85	60	18·287
229 66	70	21·336
262·47	80	24·384
295·28	90	27·431
328·09	100	30·479
3280·90	1000	304·787

TABLE II

Miles and Kilometres Conversion Table

Miles.		Kilometres.
·621	1	1·609
1·242	2	3·219
1·863	3	4·824
2·486	4	6·437
3·105	5	8·047
3·726	6	9·656
4·347	7	11·265
4·968	8	12·879
5·589	9	14·484
6·213	10	16·093
6·831	11	17·702
7·453	12	19·312
8·074	13	20·921
8·695	14	22·530
9·316	15	24·139
9·937	16	25·748
10·558	17	27·357
11·179	18	28·966
11·800	19	30·575
12·426	20	32·185
18·641	30	48·279
24·854	40	64·373
31·069	50	80·466
37·283	60	96·559
43·497	70	112·652
49·710	80	128·746
55·924	90	144·839
62·13	100	160·934

TABLE III

Metres per Second and Miles per Hour

Metres per Second.		Miles per Hour.
0·447	1	2·24
0·894	2	4·47
1·341	3	6·71
1·788	4	8·95
2·235	5	11·80
2·682	6	13·42
3·129	7	15·66
3·576	8	17·90
4·023	9	20·14
4·47	10	22·37
4·92	11	24·61
5·36	12	26·84
5·81	13	29·08
6·26	14	31·32
6·71	15	33·55
7·15	16	35·97
7·60	17	38·03
8·05	18	40·27
8·49	19	42·51
8·94	20	44·74
9·39	21	46·98
9·83	22	49·21
10·28	23	51·45
10·73	24	53·69
11·17	25	55·92
11·62	26	58·16
12·07	27	60·34
12·52	28	62·64
12·96	29	64·87
13·41	30	67·11
17·88	40	89·48
22·35	50	111·85
26·82	60	134·22
31·29	70	156·59
35·76	80	198·96
40·23	90	201·33
44·70	100	223·70

TABLE IV

Velocity and Pressure of the Wind

Kg. per Square Metre. ($R = \cdot075\ V^2$).		Lbs. per Square Foot. ($R = \cdot003\ V^2$).	
Metres per Second.	Kg.	Miles per Hour.	Lbs.
1	·075	1	·003
2	·300	2	·012
3	·675	3	·027
4	1·200	4	·048
5	1·875	5	·075
6	2·700	6	·108
7	3·675	7	·147
8	4·800	8	·192
9	6·075	9	·243
10	7·500	10	·300
11	9·075	15	·675
12	10·800	20	1·200
13	12·685	25	1·875
14	14·700	30	2·700
15	16·875	35	3·675
16	19·200	40	4·800
17	21·675	45	6·075
18	24·300	50	7·500
19	27·075	60	10·800
20	30·000	70	14·700
25	46·875	80	19·200
30	67·500	90	24·300
35	91·875	100	30·000
40	120·000	150	67·500
50	187·500	200	120·000

TABLE V

DEGREES, GRADIENTS, AND SINES

Degrees.	Gradients.	Sines.
1	1 in 57	·0175
1·91	„ 30	·0333
2	„ 28·5	·0349
2·29	„ 25	·0400
2·87	„ 20	·0500
3	„ 19	·0523
4	„ 14·3	·0698
4·78	„ 12	·0833
5	„ 11·4	·0872
5·73	„ 10	·1000
6	„ 9·8	·1045
6·38	„ 9	·1111
7	„ 8·1	·1219
7·18	„ 8	·1250
8	„ 7·2	·1392
8·22	„ 7	·1430
9	„ 6·4	·1564
9·6	„ 6	·1667
10	„ 5·7	·1736
11	„ 5·2	·1908
11·53	„ 5	·2000
12	„ 4·8	·2079
13	„ 4·5	·2250
14	„ 4·1	·2419
14·48	„ 4	·2500
15	„ 3·9	·2588
16	„ 3·6	·2756
17	„ 3·4	·2924
18	„ 3·2	·3090
19	„ 3·1	·3256
19·45	„ 3	·3333
20	„ 2·9	·3420

SYMBOLS AND NOTATION USED IN THE PRESENT WORK

S . . . Area (in sq. metres).
p . . . Total air-pressure on normal flat plane (in kilos.) $= 0{\cdot}08\,SV^2$ (§ 1).
V . . Relative wind velocity (metres per second) $= \sqrt{\dfrac{P}{KSi}}$.
q . . . Total air-pressure on inclined flat plane (in kilos.) $= kSV^2i$ (§ 2).
i . . . Angle of incidence (expressed as decimal fraction)—§§ 2, 3.
k . . . Coefficient of air resistance on flat planes. For value see footnote, p. 4.
Q . . Total air-pressure on inclined curved plane (in kilos.) $= KSV^2i$ (§§ 3, 5).
K . . Lifting efficiency ; coefficient of air resistance on curved plane of good aspect ratio $= 0{\cdot}4$ (§ 3).
P . . . Weight of aeroplane (in kilos.) $= F = KSV^2i$ (§ 5).
F . . . Lift ; vertical component of air resistance $= P = Q$.
V_R . . Normal speed (§§ 7, 8).
i_R . . Normal angle of incidence (§§ 7, 8).
R . . Total air resistance on an aeroplane (§ 11).
t . . . Drift of an aeroplane, or thrust (in kilos.) $= h + p = P\left(i + \dfrac{1}{f^2 i}\right)$ (§ 11).
h . . . Drift of the planes alone, or active resistance $= Pi$ (§ 11).
s . . . Detrimental surface ; head resistance of an aeroplane $= \dfrac{P}{f^3 i}$ (§ 11).
i_1 . . . Optimum angle $= \dfrac{1}{f}$ (§ 12).
t_1 . . . Minimum thrust $= 2Pi_1$ (§ 12).
V_1 . . Most efficient speed (§ 12).
f . . . Fineness $= \dfrac{1}{i_1}$ (§ 14).
T_u . . Useful power $= \dfrac{Vt}{75}$ (in horsepower)—§§ 18 and 24.
T_m . . Motive power $= 2T_u$ (§§ 19 and 24).
i_e . . Economical angle of incidence $= i_1\sqrt{3}$ (§ 20).
V_e . . Economical speed (§§ 20 and 27).

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